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Explaining the physical origin of Båth's law

Jure Žalohar

University of Ljubljana, Faculty of Natural Sciences and Engineering, Department of Geology, Aškerčeva 12, SI-1000 Ljubljana, Slovenia

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ABSTRACT

Båth's law is one of the three well-known scaling laws for earthquakes. It states that the difference in magnitudes of the mainshock and its largest aftershock is approximately constant, independent of the magnitude of the mainshock. Despite the progress in understanding the nature of Båth's law, the question of whether this law has a physical basis, or is simply a consequence of basic statistical features of aftershock sequences, has remained controversial. In this article we show that Båth's law can be derived within the Cosserat continuum theory from equations describing fault interaction. Our equations can describe both (1) the interacting mainshocks and aftershocks, and (2) the interacting foreshocks and mainshocks. We also derive (1) spatial extension of Båth's law to the normalized distance between the locations of the interacting mainshocks and aftershocks (or foreshocks and mainshocks), and (2) temporal extension of Båth's law to the difference between the time of the interacting mainshocks and aftershocks (or foreshocks and mainshocks).

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1. Introduction

It is well known that an earthquake of large magnitude is usually followed by a sequence of smaller earthquakes called aftershocks (e.g. Nanjo et al., 2007), which are caused by stress transfer (diffusion) induced either by mainshock or a viscous relaxation process (e.g., Helmstetter and Sornette, 2002; Shcherbakov and Turcotte, 2004). Aftershocks occur close to the focus of the mainshock, gradually decreasing in activity with increasing time over many months and even years. The behavior of aftershock sequences has been studied statistically in detail by numerous authors who derived several empirical scaling laws (Utsu, 1961; Kisslinger and Jones, 1991; Kisslinger, 1996). The three best known are Gutenberg-Richter's law, modified Omori's law and Båth's law. Gutenberg-Richter's law has been found to be applicable to the aftershock frequency-magnitude statistics, while the temporal decay of aftershock sequences in many cases satisfies modified Omori's law. Båth's scaling law states that it is a good approximation to assume that the difference in magnitude between mainshock, m_1, and its largest aftershock, m_2, is constant, $m_1 - m_2 = const.$, independent of the magnitude of the mainshock (Båth, 1965). A number of extensive studies of the statistical variability of Båth's law have been carried out (Vere-Jones, 1969; Shcherbakov and Turcotte, 2004; Saichev and Sornette, 2005; Kisslinger and Jones, 1991; Console et al., 2003; Helmstetter and Sornette, 2002; Vere-Jones et al., 2006; Lombardi, 2002). Despite the progress in understanding the nature of Båth's law, the question of whether this law has a physical basis, or is simply a consequence of basic statistical features of aftershock sequences, has remained controversial. In early studies, Vere-Jones (1969) pointed out that even without any physical basis, the distribution of the difference between the magnitudes of the largest and second largest events in an arbitrary sequence of events should be independent of the number of events in the sequence. This hinted some form of statistical origin of the law. Lombardi (2002) and Console et al. (2003) explored this possibility in a much greater depth, and concluded it was still possible that some additional physically-based component was needed to explain the observations. Even more recently, Helmstetter and Sornette (2002), and Saichev and Sornette (2005) have made detailed studies of the general branching process, called the Epidemic Type Aftershock Sequence (ETAS) model of triggered seismicity. The ETAS model is a multigenerational model in which aftershocks from one earthquake trigger their own aftershock sequences through multiple generations of earthquakes. The frequency and magnitude distributions of the triggered events, while still fundamentally based on Omori's law and Guttenberg-Richter's law, have more complex formulations to better represent the statistical distributions seen in earthquakes catalogs. Within these models Helmstetter and Sornette (2003) and Saichev and Sornette (2005) established conditions under which Båth's law appears. In addition, Helmstetter and Sornette (2003) pointed out that the selection process of aftershock sequence as a subset of the whole seismic catalog plays a significant role in calculating the difference in magnitudes between the largest and

E-mail address: jure.zalohar@guest.arnes.si.

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the second largest event. They argued that this difference is controlled not only by the magnitude scaling but also by the aftershock productivity.

Vere-Jones et al. (2006) described another general basis for Båth's law without the need for a special physical mechanism, while Shcherbakov and Turcotte (2004) developed a modified version of Båth's law on a more physical basis. They introduced the notion of the inferred "largest" aftershock from an extrapolation of the Gutenberg-Richter's frequency-magnitude statistics of the aftershock sequence of a given mainshock. The scaling associated with modified Båth's law implies that the stress transfer responsible for the occurrence of aftershocks is a self-similar process. They also estimated the partitioning of energy during a mainshock-aftershock sequence and found that about 96% of the energy dissipated in a sequence is associated with the mainshock, while the rest is associated with the aftershocks. They suggested that the observed partitioning of energy could play a crucial role in explaining the physical origin of Båth's law.

In this article we describe a new approach in understanding the physical basis of Båth's law. We show that Båth's law can be derived from equations describing fault interaction in the Cosserat continuum. In the last two decades, considerable progress in understanding the faulting process by applying the Cosserat continuum theory showed that the classical continuum theory is incapable of properly describing geological faulting (e.g., Twiss et al., 1991, 1993; Twiss and Unruh, 1998, 2007; Figueredo et al., 2004; Twiss, 2009; Žalohar and Vrabec, 2010; Žalohar, 2012). It was discovered that the movement along the faults is related in a systematic way to the global deformation and rotation fields, and to the local microrotations of blocks bounded by the fault planes. Any rotational deformation of the Earth's crust due to the slip along more or less dense networks of faults is related to a difference between the regional macrorotation of the rock mass and local microrotation of individual blocks. This difference is called the relative microrotation and results in correlation and interaction between the movements along the faults that represent the boundary of the blocks involved. The Cosserat continuum theory predicts three types of fault interactions: (1) intersecting-faults interaction (Žalohar and Vrabec, 2010; Žalohar, 2012), (2) parallel–faults interaction (this article), and (3) parallel–wedges interaction (this article). We show that in the Cosserat continuum, the parallel–faults and parallel–wedges interactions lead to Båth's law. In the first chapter, we summarize some basic ideas of the Cosserat continuum. The meaning of these basic principles is not fully discussed, so the reader should refer to the articles by Twiss et al. (1991, 1993), Twiss and Unruh (1998, 2007), Figueredo et al. (2004), Twiss (2009), Žalohar and Vrabec (2010) and Žalohar (2012). In the following chapters the mathematics behind the parallel–faults interaction is described. The presented mathematical analysis is a continuation of the article written by Žalohar (2012), who described the interaction between intersecting faults, along which the slip direction is subparallel to their common intersection direction (the wedge faulting).

2. Kinematics of the Cosserat continuum

Kinematics of the Cosserat continuum is characterized by a (micro)rotational degree of freedom $\vec{\phi}^{\text{Cosserat}}$ (Fig. 1), which is independent of the translatory motion described by the displacement field $\vec{u}$. Let $\vec{u}_1$ and $\vec{u}_2$ represent the displacements of the two neighboring blocks, and $\vec{\phi}^{\text{Cosserat}}$ represents their microrotations. The size of the blocks, L, is equal to the distance between their centroids, and also represents a *characteristic length* of the rotating

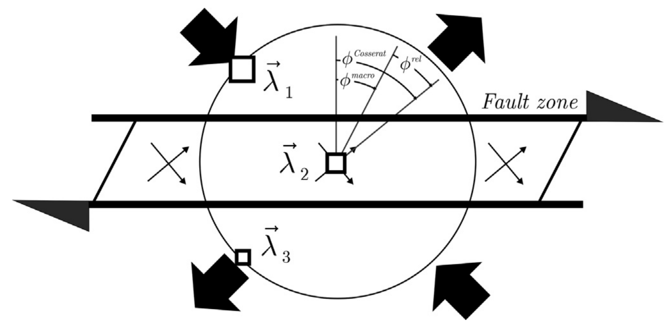

Fig. 1. Micro- and macro-rotations in a Cosserat continuum (after Žalohar, 2012). $\vec{\phi}^{\text{macro}}$ represents the large-scale regional rotation related to faulting (= axial vector of the deformation gradient tensor). $\vec{\phi}^{\text{Cosserat}}$ is the Cosserat microrotation of individual segments of the Cosserat continuum. $\vec{\phi}^{\text{rel}}$ is the relative microrotation, which is defined as the difference between the macrorotation and the Cosserat microrotation. $\vec{\lambda}_1$, $\vec{\lambda}_2$, and $\vec{\lambda}_3$ are kinematic axes of the instantaneous deformation tensor (the eigenvectors of this tensor).

blocks in the Cosserat continuum. The distance between the center of microrotation and the boundary (fault) between the two blocks is $L/2$. The relative motion on the boundary between the blocks (on the fault) is:

$$\vec{u}_{rel} = \vec{u}_{12} + \frac{L}{2}\left(\vec{n} \times \vec{\phi}_{12}\right) = \vec{u}_{12} - \frac{L}{2}\left(\vec{\phi}_{12} \times \vec{n}\right), \tag{1}$$

where $\vec{u}_{12} = \vec{u}_2 - \vec{u}_1$ is the relative translation of the first block with respect to the second, and $\vec{\phi}_{12} = 2\vec{\phi}^{\text{Cosserat}}$ is the relative micrototation of the first block with respect to the second. $\vec{n}$ is the unit normal to the fault splitting the blocks apart. Note that in Equation (1) the center of rotation coincides with the mass center of the blocks. In the Cosserat continuum the corresponding strain measures are the *Cosserat strain tensor* $\mathbf{e}$ and the *torsion-curvature tensor* $\boldsymbol{\kappa}$ (Forest, 2000; Forest and Sievert, 2003; Forest et al., 1997, 2000, 2001):

$$\begin{aligned}\mathbf{e} &= \mathbf{u} - \mathbf{W}^C = \vec{u} \otimes \vec{\nabla} + \underline{\varepsilon}\,\vec{\phi}^{\text{Cosserat}} = \frac{\partial u_i}{\partial x_j} + \varepsilon_{ijk}\phi_k^{\text{Cosserat}}, \\ \boldsymbol{\kappa} &= \vec{\phi}^{\text{Cosserat}} \otimes \vec{\nabla} = \frac{\partial \phi_i^{\text{Cosserat}}}{\partial x_j}.\end{aligned} \tag{2}$$

We have also introduced the deformation gradient tensor (or instantaneous macrodisplacement gradient) $\mathbf{u} = u_{ij} = \partial u_i/\partial x_j = \vec{u} \otimes \vec{\nabla}$, and the Cosserat microrotation tensor $\mathbf{W}^C = -\underline{\varepsilon}\,\vec{\phi}^{\text{Cosserat}}$. The symmetric part $\mathbf{u}^{(S)}$ of the deformation gradient tensor defines the macrostrain, which describes the deformation of the line joining the centroids of the two blocks, while the antisymmetric part $\mathbf{u}^{(A)}$ defines the instantaneous macrorotation, which contributes to the rotation of the line joining the centroids (e.g., Twiss and Unruh, 2007). The eigenvalues and eigenvectors of the $\mathbf{u}^{(S)}$ are denoted by λ_1, λ_2, λ_3 and $\vec{\lambda}_1$, $\vec{\lambda}_2$, $\vec{\lambda}_3$. Because the deformation gradient tensor can be decomposed into the symmetric and skew-symmetric parts $\mathbf{u} = \mathbf{u}^{(S)} + \mathbf{u}^{(A)}$, the Cosserat strain tensor can be written as:

$$\mathbf{e} = \mathbf{u}^{(S)} + \mathbf{u}^{(A)} - \mathbf{W}^C = \mathbf{u}^{(S)} + \mathbf{A}, \tag{3}$$

with $\mathbf{e}^{(S)} = \mathbf{u}^{(S)}$ and $\mathbf{e}^{(A)} = \mathbf{A}$. We have introduced the *relative microrotation tensor*

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