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Strain analysis from objects with a random distribution: A generalized center-to-center method

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ABSTRACT

Existing methods of strain analysis such as the center-to-center method and the Fry method estimate strain from the spatial relationship between point objects in the deformed state. They assume a truncated Poisson distribution of point objects in the pre-deformed state. Significant deviations occur in nature and diffuse the central vacancy in a Fry plot, limiting the its effectiveness as a strain gauge. Therefore, a generalized center-to-center method is proposed to deal with point objects with the more general Poisson distribution, where the method outcomes do not depend on an analysis of a graphical central vacancy. This new method relies upon the probability mass function for the Poisson distribution, and adopts the maximum likelihood function method to solve for strain. The feasibility of the method is demonstrated by applying it to artificial data sets generated for known strains. Further analysis of these sets by use of the bootstrap method shows that the accuracy of the strain estimate has a strong tendency to increase either with point number or with the inclusion of more pre-deformation nearest neighbors. A poorly sorted, well packed, deformed conglomerate is analyzed, yielding strain estimate similar to the vector mean of the major axis directions of pebbles and the harmonic mean of their axial ratios from a shape-based strain determination method. These outcomes support the applicability of the new method to the analysis of deformed rocks with appropriate strain markers.

1. Introduction

The mutual spatial arrangement of objects including sedimentary clasts, fossils, ooids, polycrystal aggregates, porphyroblasts, in rocks is frequently treated as a passive strain marker to determine the finite strain of the host rock, in two ways: the center-to-center method (Ramsay, 1967) and the all object–object separations or Fry method (Fry, 1979; Hanna and Fry, 1979). The nearest neighbor objects in the pre-deformed state need be known for the former method, but not necessarily for the latter method that has found a wider application for this reason. As both methods explicitly or implicitly assume, the study objects are randomly distributed, well sorted and well packed in the pre-deformed state, so that they would define in the Fry plot a blank fabric ellipse that has a similar orientation and a similar axial ratio to the strain ellipse (Fig. 1a–c). For instance, Fry (1979) considered a truncated Poisson distribution of point objects with its associated randomness, sorting and packing properties. The deviation of one or more of these properties from the assumptions thus largely accounts for the uncertainty of strain estimates in practice. Accordingly, modifications to the two methods have since been made to visually or numerically enhance recognition of the fabric ellipse, including the Delaunay triangulation method (Mulchrone, 2002, 2013) for the former, and the normalized Fry method (Erslev, 1988), the enhanced normalized Fry method (Erslev and Ge, 1990), the modified normalized Fry method (McNaught 1994, 2002), the point-count density method (Waldron and Wallace, 2007), the retro-deformation method (Lisle, 2010), the maximum likelihood function method (Shan and Xiao, 2011), the image analysis method (Sampath and Srivastava, 2011), and so forth for the latter.

Despite these modifications, it is still a challenge to use objects with a distribution that severely violates the above-mentioned assumptions, because the estimated strains using these methods may deviate significantly from the actual finite strain. Deviations of actual spatial strain markers from the characteristics of an ideal population tends to diffuse the quality of the needed vacancy or gap for the strain determination. For example, objects may be well sorted but loosely packed (Fig. 1d–f), or badly sorted but closely packed (Fig. 1g–i), creating ill defined vacancies that preclude useful strain measurement. Although as described, a variety of methods exist for attempting to resolve this issue, we believe that

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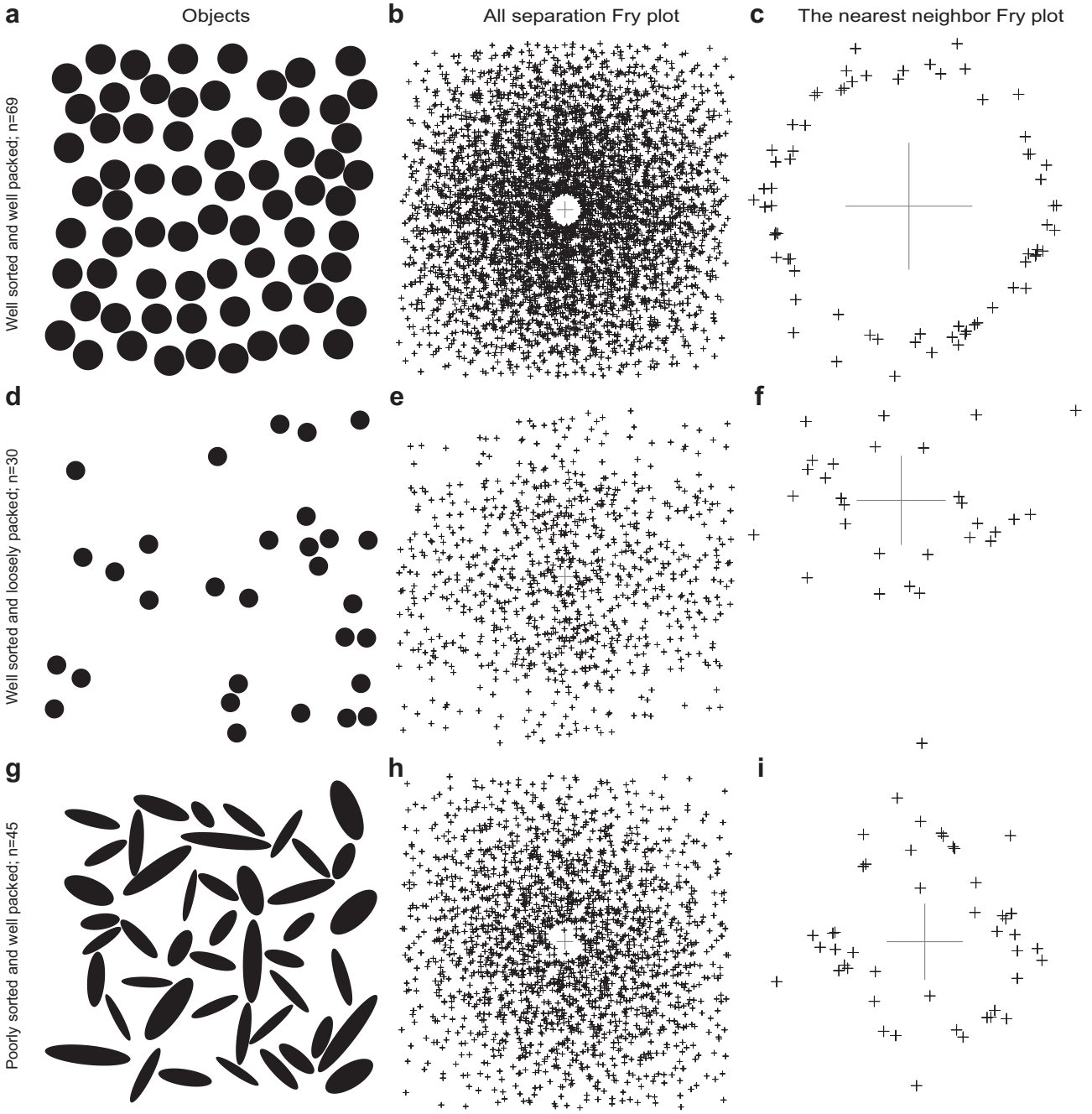

Fig. 1. Presentations of objects with three contrasting distributions, well sorted and well packed (a–c), well sorted and loosely packed (d–f), and poorly sorted and well packed (g–i). These objects are artificially generated in the non-deformed state. In Fry plot, the small black "plus" symbols are either all object–object separation points (b, e and h) or the first-nearest neighbor points (c, f and i), and the large gray "plus" symbols represent the origin of the plot, with a similar size. The vacancy that the nearest neighbor points define in the central part of the Fry plot appears to have a circular shape in the first case (c), a square shape in the second case (f), and a rectangle or elliptical shape in the third case (i).

treating all points as members of a Poisson distribution rather than as a truncated Poisson distribution is the key to successful issue resolution.

2. Principles of method

The Poisson process is widely used to describe the occurrences of the centers of objects or point objects in rocks for two-dimensional strain analysis (e. g. Fry, 1979; Shan and Xiao, 2011). In this discrete process, if objects are supposed to happen one by one, then each individual object happens independently of the relative timing and of existing objects. Let us consider a case of N Poisson objects in a bounded region $\boldsymbol{W}$ and involving no deformation (Fig. 2a). The possibility mass function of having a number of $k\,(1 \le k < N)$ objects within a certain circular subregion $\boldsymbol{V}\,(V \in W)$ is calculated in the following expression:

$$P(k,\lambda,r) = \frac{(\lambda\pi r^2)^k e^{-\lambda\pi r^2}}{k!} \tag{1}$$

where r is the radius of the circular subregion, and λ is the mean density of objects, the point number N divided by the $\boldsymbol{W}$'s volume.