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Behaviour of an isolated rimmed elliptical inclusion in 2D slow incompressible viscous flow

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A B S T R A C T

For 2D linear viscous flow, it is shown that the rates of rotation and stretch of an isolated elliptical inclusion with a coaxial elliptical rim are fully determined by two corresponding scalar values. For power-law viscosity, effective viscosity ratios of the inclusion and rim to the matrix depend on orientation and the system is more complex but, in practice, the simplification with two scalar values still provides a good approximation. Finite-element modelling (FEM) is used to determine the two characteristic values across a wide parameter space for the linear viscous case, with a viscosity ratio (relative to the matrix) of the inclusion from 10^6 to 1, of the rim from 10^{-6} to 1, axial ratios from 1.00025 to 20, and rim thicknesses relative to the inclusion axes of 5 to 20%. Results are presented in a multi-dimensional data table, allowing continuous interpolation over the investigated parameter range. Based on these instantaneous rates, the shape fabric of a population of inclusions is forward modelled using an initial value Ordinary Differential Equation (ODE) approach, with the simplifying but unrealistic assumption that the rim remains elliptical in shape and coaxial with respect to the inclusion. However, comparison with accurate large strain numerical experiments demonstrates that this simplified model gives qualitatively robust predictions and, for a range of investigated examples, also remarkably good quantitative estimates for shear strains up to at least $\gamma = 5$. A statistical approach, allowing random variation in the initial orientation, axial ratio and rim viscosity, can reproduce the characteristic shape preferred orientation (SPO) of natural porphyroclast populations. However, vorticity analysis based on the SPO or the interpreted stable orientation of inclusions is not practical. Varying parameters, such as inclusion and rim viscosity, rim thickness, and power law-exponents for non-linear viscosity, can reproduce the range of naturally observed behaviour (e.g., back-rotation, effectively stable orientations at back-rotated angles, a cut-off axial ratio separating rotating from stable inclusions) even for constant simple shear and these features are not uniquely characteristic of the vorticity of the background flow.

1. Introduction

Inclusions in deformed rocks have been extensively studied because they provide potential markers for determining the kinematics of flow and the amount of deformation. The simplest examples to measure and analyse are those with an approximately elliptical cross-section in the plane of observation, for which an axial ratio and orientation are readily defined. Appropriate analytical solutions exist for the stretch and rotation of cylinders with elliptical cross-sections in 2D and ellipsoids in 3D if the materials are linear elastic or linear viscous and if the interface between inclusion and matrix is coherent (Muskhelishvili, 1953; Eshelby, 1957, 1959; Bilby et al., 1975, 1976; Bilby and Kolbuszewski, 1977; Schmid and Podladchikov, 2003; Mulchrone and Walsh, 2006; Jiang, 2007, 2012). However, natural observations on inclusion systems in many cases are not in good agreement with the predictions of these analytical solutions (e.g., Pennacchioni et al., 2001; ten Grotenhuis et al., 2002, 2003; Johnson et al., 2009) and clearly another factor must have a significant influence. It has been proposed that this may be the effect of an imperfectly bonded or slipping boundary (Ildefonse and Mancktelow, 1993; Arbaret et al., 2001; Marques and Coelho, 2001; Pennacchioni et al., 2001; Ceriani et al., 2003; Marques and Bose, 2004; Schmid and Podladchikov, 2004, 2005; Marques et al., 2005a; Johnson et al., 2009). It is this effect that is analysed here, using the basic 2D geometry of an isolated inclusion of elliptical cross-section with a thin, initially elliptical and coaxial weaker rim, embedded in an effectively infinite matrix. Rotation and stretching rates are only determined for the stronger inclusion and not for the surrounding rim, which

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with increasing strain can deform into quite complex shapes. The flow is taken to be slow (laminar flow at low Reynolds number) and the rheology of all three components (inclusion, rim and matrix) to be homogeneous, isotropic, incompressible and linear viscous, with the effect of non-linear (power-law) viscosity investigated for specific examples.

Published work, especially that of Schmid and Podladchikov (2005) and Johnson et al. (2009), already covers aspects of the behaviour, but the system has not been fully analysed and studies were largely restricted to the end-member case of a very strong or strictly rigid inclusion. Here a very wide range of parameters is considered for the deformable linear viscous case (viscosity ratio of inclusion to matrix from 10^6 to 1, viscosity ratio of rim to matrix from 10^{-6} to 1, axial ratios from 1.00025 to 20, rim thickness relative to the inclusion axes from 5 to 20%). Analytical considerations combined with symmetry arguments allow the problem to be simplified to the determination of two scalar parameters. However, no analytical solution is currently available to determine these parameters and the approach here is numerical, using finite-element modelling (FEM). A multi-dimensional data grid is developed to summarize the numerical results and to allow interpolation of the instantaneous stretching and rotation rates continuously across this very wide range of geometries and relative viscosity values. The data grid is provided in the Supplementary material, together with MATLAB scripts necessary to generate the figures presented in this paper. Using an initial value Ordinary Differential Equation (ODE) approach, interpolation of the instantaneous rates also allows the progressive stretch and rotation of a rimmed elliptical inclusion in 2D viscous flow to be determined and plots of axial ratio against orientation (R_f/ψ plots) for populations of inclusions to be generated. Plots with a pre-defined statistical variation in the controlling parameters can be directly compared to measurements of natural populations of inclusions in deformed rocks (Pennacchioni et al., 2001; Mancktelow et al., 2002; Johnson et al., 2009), for which the parameters are also likely to vary for the population as a whole. Predictions from this relatively simple model for individual inclusions are also compared to large strain numerical models in which the rim is deformed into more complex shapes similar to those observed in natural mantled porphyroclast systems (e.g., Passchier and Simpson, 1986; Passchier and Sokoutis, 1993; Passchier and Trouw, 2005).

2. Previous work

A summary of the quite extensive previous work on isolated inclusions with coherent boundaries is already given for the 2D case in Mancktelow (2011) and for 3D in Jiang (2007, 2012). An important result of Eshelby (1957) is that the stress and strain within a homogeneous elliptical (2D) or ellipsoidal (3D) inclusion embedded in a linear elastic matrix is uniform if the far field strain is uniform. The inclusion will therefore maintain its elliptical or ellipsoidal shape during deformation. Through the correspondence principle (e.g., Biot, 1965), Eshelby's result can be directly translated to linear viscous rheology (e.g., Bilby and Kolbuszewski, 1977; Schmid and Podladchikov, 2003). Extension of this work has shown that inclusions of more general rheology also deform homogeneously, provided they are embedded in a linear viscous matrix (Goddard and Miller, 1967; Roscoe, 1967; Bilby and Kolbuszewski, 1977; Schmid and Podladchikov, 2003; Mancktelow, 2011). However, this is not true when (1) the matrix is non-linear (e.g., Gilormini and Montheillet, 1986; Gilormini and Germain, 1987; Gilormini and Michel, 1998), (2) the inclusion is imperfectly bonded to the matrix (e.g., Mura et al., 1985; Tsuchida et al., 1986; Mura, 1987; Hashin, 1991; Gao, 1995; Shen et al., 2001; Xu et al., 2011), or (3) the elliptical inclusion is surrounded by a rim or "mantle" of different material (e.g., Kenkmann, 2000; Schmid and Podladchikov, 2004, 2005).

Schmid and Podladchikov (2004) proposed that the rotational behaviour of a rigid inclusion with a very weak rim is conceptually equivalent to that of a void of constant shape. Based on their earlier analytical solution for an isolated deformable viscous inclusion (Schmid and Podladchikov, 2003), the rotation rate of a rigid inclusion with an incoherent boundary embedded in an infinite linear viscous matrix undergoing slow simple shear flow at shear strain rate $\dot{\gamma}$ was given as

$$\frac{\dot{\psi}}{\dot{\gamma}} = -sin(\psi)^2\frac{R+1}{R} + \frac{1}{2R}, \tag{1}$$

where R is the axial ratio of the inclusion, and ψ is the angle of the long axis of the inclusion to the shear direction. This equation can be recast in double angles as

$$\frac{\dot{\psi}}{\dot{\gamma}} = \frac{1}{2}\left(\frac{R+1}{R}\cos(2\,\psi) - 1\right). \tag{2}$$

Mulchrone (2007) developed an alternative analytical solution for the rotation rate of a rigid inclusion with a slipping interface,

$$\frac{\dot{\psi}}{\dot{\gamma}} = \frac{1}{2}\left(\frac{R+1}{R-1}\cos(2\,\psi) - 1\right), \tag{3}$$

reformatted to use the same symbol conventions as in Eqs. (1) and (2). Setting the viscosity ratio of inclusion to matrix equal to zero in the more general solutions of Bilby and Kolbuszewski (1977) (e.g., Eq. (27) in Mancktelow, 2011) immediately establishes that Eq. (3) is identical to the solution for the rotation rate of an incompressible void (i.e., one that changes shape but not area). The conceptual models attributable to Eqs. (2) and (3) are therefore distinctly different. The result of Mulchrone (2007) implies that the rotation rate of a rigid inclusion with a slipping interface is identical to that of an incompressible but deformable void, whereas that of Schmid and Podladchikov (2004) corresponds to a rigid or "equivalent void" that is not allowed to change its shape. However, in practice the numerical experiments of Schmid and Podladchikov (2004, 2005) modelled the geometry as shown in Fig. 1, for which the rim is deformable. For this geometry, Johnson et al. (2009)

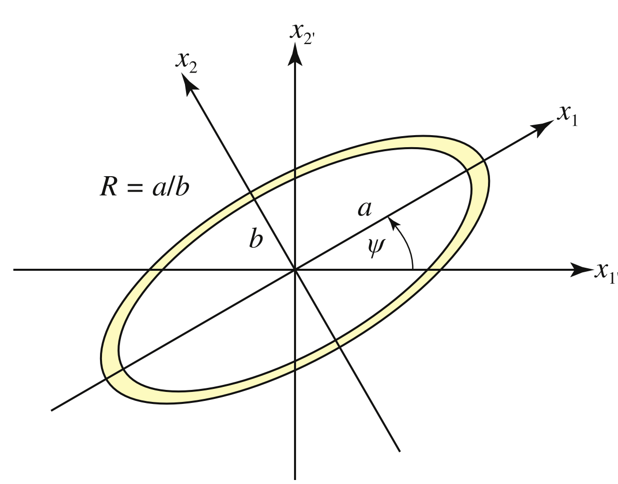

Fig. 1. 2D cross-sectional geometry of the rimmed inclusion system. The rim consists of the zone between the elliptical inclusion and a concentric and coaxial ellipse with the same axial ratio R but larger axes: a 10% rim (as shown) corresponds to axes for the outer ellipse that are 1.1 times the axes of the inclusion ellipse. The inclusion and rim are cylindrical in the third dimension.