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Research Paper

Analytical study of thermal spreading resistance in curved-edge heat spreader

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HIGHLIGHTS

- Heat spreading/conduction equation is derived in general curvilinear coordinates.
- Maxwell transform is used to map the geometry of curved-edge heat spreader.
- Thermal spreading resistance of a curved-edge heat spreader is calculated.

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ABSTRACT

Because pieces of microelectronic devices are made in a wide variety of scales and shapes, heat must flow through them in spreading or constriction forms. Two heat flow conditions are primarily responsible for thermal spreading resistance: heat flowing from one solid to another with different cross-sectional areas (the primary focus of past studies); and heat flowing through a conductive solid with variable cross-sectional area. In this study, both conditions are considered simultaneously. The equation governing heat spreading is derived in the general curvilinear coordinate system. The Maxwell coordinate system is used as a special case to map the irregular geometry from Cartesian coordinates to the boundary-fitted curvilinear coordinate system. Temperature distribution and spreading resistance are then estimated by solving the equation governing heat conduction. A generalized thermal resistance is then introduced to evaluate the impact of variable cross-sectional area and heat source length on heat spreading. Finally, the effects of heat source length and the Biot number on spreading resistance are investigated.

1. Introduction

THERMAL spreading resistance appears whenever heat cannot be transferred uniformly from a heat source to a larger heat sink. This resistance prevents an efficient flow of heat from the source to the ambient. This phenomenon is an increasingly important topic in microelectronics thermal management, especially for faster and high powered devices. Total thermal resistance comprises more than 50% of speeding resistance; therefore, a good understanding of spreading resistance is essential to understanding cooling in the microelectronics field. Despite its importance, many parameters affecting spreading resistance are poorly understood. Therefore, many recent studies have focused on understanding spreading resistance under various physical and geometrical conditions.

Ellison [1] solved the 3-D rectangular heat conduction equation used to compute the maximum spreading resistance of nonunity aspect ratio sources on square plates. He reported dimensionless solutions for maximum and source-averaged thermal spreading resistances. Sadeghi et al. [2] presented an analytical approach to the calculation of spreading resistance for various geometries. Although their work was based on the generalization of the solution of isoflux elliptical sources on a half-space, they presented results for both isoflux and isothermal conditions. Dong et al. [3] analyzed the thermal spreading effect in flip chip and face-up chip structures in LED packaging. Furthermore, Li et al. [4,5] conducted experimental studies on the thermal performance of high-power LEDs under new thermoelectric cooler cooling system. Their studies demonstrate that their proposed cooling system has good performance. Rahmani and Shokouhmand [6] numerically calculated the spreading resistance of silicon for both heat flux tubes and half-space models in order to assess the effects of temperature-dependent conductivity, the shape of the contact surface, and the size of the contact and boundary conditions. Guan et al. [7]

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Nomenclature

a	length of the spreader top surface (m)
A	Area (m^2)
Bi	Biot number, dimensionless quantity
b	length of the spreader bottom surface (m)
c,C	length of the heat source (m), summation constant
cte	constant value
F	complex function
$\bar{f}$	complex function after mapping
h	convection heat transfer coefficient (W/m^2 K)
k	thermal conductivity (W/m K)
n	unit normal vector, positive in the outward direction
Q	heat flow rate (W)
q''	heat flux (W/m^2)
R	thermal resistance (K/W)
T	temperature (K)
x, y, z	Cartesian coordinates (m)
$\bar{x}$, $\bar{y}$, $\bar{z}$	Cartesian coordinates after transformation (m)
w, z	complex variables

Greek symbols

δ	height of heat spreader (m)
θ	variable temperature substitution, (K)
λ	eigenvalues of the Fourier expansion
ξ,η	Maxwell curvilinear coordinates (m)
$\bar{\xi}$, $\bar{\eta}$	Maxwell curvilinear coordinates after transformation (m)
Φ	series summation from $n = 1$ to $n = N$

Subscripts

a	spreader top surface length
c	heat source
cte	constant value
$g1$	generalized value, isothermal sink case
$g2$	generalized value, convective sink case
max	maximum value
n	index for summations
r	reference value
s	heat sink, spreading
0	summation constant subscript
∞	fluid temperature, convective sink case
1	isothermal sink case
2	convective sink case

developed an analytical solution for calculating spreading resistance when the same footprint of a heat source is assumed to be located on a three-layer substrate. They also proposed an approximate approach to estimate the dimensions of unequal substrate layers. Rahmani and Shokouhmand [8] used the Kirchhoff transformation to determine the thermal spreading resistance of semiconductor materials with temperature-dependent conductivities. The boundary conditions of the heat sink were assumed to be of the first and second kind (constant temperature and heat flux, respectively). The results of their study showed that the temperature dependence of thermal conductivity in these materials should be considered in order to find thermal spreading resistance because the thermal conductivities of these materials have a high sensitivity to temperature fluctuation. In addition, Bagnall et al. [9] used the Kirchhoff transform to linearize the heat conduction equation of spreading resistance with boundary Conditions of the third kind (convection) in the sink plane. Cruz et al. [10] calculated the thermal spreading resistance of aluminum heat sink with ammonia gas using simulated annealing, unified particle swarm and spiral algorithms. Yang et al. [11] conducted experimental and numerical studies to evaluate the substrate thickness and utilizing materials with high lateral thermal conductivities on the thermal spreading. Feng and Xu [12] proposed a three-dimensional analytical solution of thermal spreading resistance in cubic heat spreaders. Li and Lucang [13] experimentally studied the effect of thermal spreading resistance on the thermal performance of a copper–water flat heat pipe. Wang [14] conducted a numerical study of thermal performance in a heat sink-heat pipes thermal module and calculated the thermal spreading resistance of system. McWaid and Marschall [15] experimentally determined the thermal spreading resistance for plastic and elastic contact. Li et al. [16] experimentally investigated the effects of mechanical parameters such as maximum landing force and channel gap on the performance of magnetorheological fluid damper. They also compared results with finite element simulation. Li et al. [17] experimentally proved that the ultrasonic vibration in flip chip bonding results in the generation of dislocations, and the atomic diffusion can be activated more easily along the dislocation lines which perform the fast diffusion channels. Muzychka et al. [18] studied two-layer orthotropic composite structures with interfacial resistance, and estimated the spreading resistance for a finite rectangular heat source of uniform strength which was arbitrarily located on a substrate. The solution was also applied to the calculation of spreading resistance of gallium nitride (GaN) structures. Moreover, Muzychka [19] obtained a solution for composite disks, and then used contact conductance to account for interfacial resistance. Gholami and Bahrami [20] developed an analytical model for the temperature rise inside a rectangular composite structure with multiple heat inputs and outputs on its top and bottom surfaces. They employed numerical software to validate the results of the proposed model. Bagnall et al. [21] developed an analytical solution to temperature distribution in an anisotropic rectangular plate with interfacial conductance subjected to isoflux heat sources, and convective boundary conditions in the sink base. They also applied the solution to study the (GaN)-based epitaxial layers in a realistic packaging configuration. Recent experimental works have been focused on the thermal spreading resistance effects on the thermal performance of high power InGaN LED module, pico projectors and Condensing heat exchangers [22–24]. Zoha Azizi et al. [25] studied thermal performance and friction factor of a cylindrical microchannel heat sink cooled by Cu–water nanofluid. They proposed two correlations for the prediction of the Nusselt number and friction factor for the attempted nanofluid in the microchannel.

From the perspective of thermal management, attention has been focused on minimizing the sizes and maximizing the reliability of system components subjected to high thermal loads. Researchers usually use many techniques, methods and transforms based on such well-known coordinate systems as the Cartesian coordinate system to assess the effects of thermo-physical and geometrical parameters on thermal spreading resistance. But irregular geometry cannot easily be studied analytically in conventional coordinate systems. Therefore, there is a need for a new, body-fitted curvilinear coordinate system which will accommodate the precise description of irregular shapes like curved edge plates. Curved geometry is common in the latest microelectronic component such as curved screen in the TV, smartphone and smart