Applied Thermal Engineering 104 (2016) 577–586

Contents lists available at ScienceDirect

Applied Thermal Engineering

journal homepage: www.elsevier.com/locate/apthermeng

Research Paper

Temporal kurtosis of dynamic pressure signal as a quantitative measure of combustion instability

Won Joon Song [a], Dong-Jin Cha [b,*]

[a] Institute for Fusion Technology for Production, Hanbat National University, San 16-1, Dukmyung-dong, Yuseong-gu, Daejeon 305-719, Republic of Korea
[b] Department of Building and Plant Engineering, Hanbat National University, San 16-1, Dukmyung-dong, Yuseong-gu, Daejeon 305-719, Republic of Korea

HIGHLIGHTS

- Kurtosis of dynamic pressure signal is used to quantify the combustion stability.
- Temporal kurtosis serves as a real-time indicator of varying combustion stability.
- Temporal kurtosis serves as a precursor of upcoming severe combustion instability.

ARTICLE INFO

Article history:
Received 25 December 2015
Revised 27 March 2016
Accepted 15 May 2016
Available online 17 May 2016

Keywords:
Combustion instability
Dynamic pressure
Temporal kurtosis
Rocket engine
Gas turbine
Aero engine

ABSTRACT

In this study, the kurtosis of dynamic pressure (DP) signal is implemented to quantify the thermoacoustic stability of a combustion process. The variation of the temporal characteristics of DP signal experiencing the three phases of thermoacoustic stability (i.e., stable, transitional, and unstable with limit cycle oscillation) was explained in terms of the temporal kurtosis (TK), the instantaneous kurtosis evaluated with a subset of DP signal. The explanation suggested that TK would be a promising parameter as a quantitative measure of combustion instability (CI). The feasibility of TK as an indicator of CI was investigated using DP data obtained from combustion tests with a rocket engine, with a lab-scale gas turbine combustor, and with a lab-scale aero engine combustor. In these feasibility studies, the time history of TKs reflected the temporal changes in combustion stability during the combustion processes successfully. It turns out that TK can serve as a real-time indicator of the change in stability condition and also as a precursor of upcoming severe instability. It is expected that the implementation of TK in quantifying the progress of thermoacoustic instability would significantly enhance the performance of commercialized monitoring systems of CI.

1. Introduction

Combustion instability (CI) of heat conversion systems including rocket engines [1], gas turbines [2], etc. can be interpreted as the thermoacoustic instability arising from feedback coupling between acoustics and heat release that excites one or more of acoustic eigenmode of the systems. Also, a thermal flux in practical energy applications such as prime mover, refrigeration, or mixture separation may cause thermoacoustic oscillation [3]. The instability is typically accompanied by violent oscillations of dynamic pressure (DP) [4]. Exposure to CI would deteriorate the performance and the life of the systems. Although much effort has been made to gain insight the phenomenon [5,6] and to control the problem either passively [7] or actively [8], no consensus on the most reasonable solution to CI has been reached due to the intrinsic complexity of the physical processes that interact with each other. DP signal is frequently monitored to characterize a combustion process. Signal processing technique, especially the frequency-domain treatment of a signal, for example, fast Fourier transform (FFT), is usually implemented for the purpose. However, the spectral analysis of DP signal is often inadequate for identifying the transition of stability condition or for predicting an upcoming instability strongly affected by the inherent nonlinearities of the phenomenon [9]. Also, the frequency-domain transform of a highly transient signal (e.g., DP signal at around the onset of CI) often loses temporal details of the signal because of a low spectral resolution inherent for short-duration data. Instead, for better identification of a combustion process, a concurrent time-domain analysis of DP signal is recommended.

Kurtosis of DP signal can be proposed as a time-domain parameter to indicate the variation of thermoacoustic stability during a combustion process. Kurtosis, a measure of the "peakedness" and

---

* Corresponding author.
E-mail addresses: udrmyarm@gmail.com (W.J. Song), djcha@hanbat.ac.kr (D.-J. Cha).

http://dx.doi.org/10.1016/j.applthermaleng.2016.05.094

"tailedness" of the statistical distribution of a given signal [10], has been widely employed to detect the failure of mechanical components [11–15]. However, to the best of our knowledge, not much attention has been paid to kurtosis of DP signal in assessing CI. Rather the quantity has been used only to identify the acoustic condition near blowout in combustors [16,17].

In this paper, a time-domain statistical parameter called temporal kurtosis (TK) is newly defined to use the statistical quantity in judging the stability condition of a combustion process. The variation of the temporal characteristics of DP signal exhibiting the transition of thermoacoustic stability was explained in terms of TK. The feasibility of using TK as a simple but efficient measure of CI was tested with experimental benchmark data.

2. Temporal kurtosis for characterizing combustion instability

Kurtosis K is defined as the ratio of the fourth moment to the squared second moment of a given continuous signal x [10]:

$$K = \frac{E(x-\mu)^4}{\left[E(x-\mu)^2\right]^2} = \frac{\mu_4}{\sigma^4} \tag{1}$$

where E is an expectation operator, μ is the mean value of the signal, μ_4 is the fourth moment about the mean value, and σ is the standard deviation. Note that the value of kurtosis is 3.0 for a random signal with Gaussian distribution (i.e., normal distribution), 1.5 for a sinusoidal signal regardless of the amplitude and the frequency, and ranges from 1.5 to 3.0 for a sinusoid embedded within Gaussian noise [18]. The two values (i.e., 1.5 and 3.0) of kurtosis would provide theoretical criteria in judging the stability condition of a combustion process, as described later.

TK, the kurtosis sequentially evaluated for partial data of a given signal, represents the instantaneous "peakedness" and "tailedness" of the statistical distribution of a given signal [10]. The collection of TKs for the entire signal reflects the time-dependent variation of both distributional features. TK of a discrete time-domain DP signal $p(t)$ for a given time duration T is described as:

$$\mathrm{TK} = \frac{\frac{1}{n}\sum_{i=1}^{n}(p_i-\bar{p})^4}{\left[\frac{1}{n}\sum_{i=1}^{n}(p_i-\bar{p})^2\right]^2} \tag{2}$$

where n is the number of DP data for the time duration T, p_i is the instantaneous DP signal value at time $t = t_i$ $(i = 1, 2, \cdots n)$, and $\bar{p}$ is the mean value of DP for the duration. The discretization of a continuous signal may lead to a little augmentation of the kurtosis values. The evaluation of TK is a simple arithmetic process without any mathematical complexities.

Thermoacoustic instability of a combustion process alters the temporal characteristics of DP signal. During stable combustion, near-normal distribution of noise components (i.e., low level background noise) is prevalent in DP signal. Eigenfrequency components are latent in the signal. The phase mismatch between the heat release rate and the acoustic pressure fluctuation just establishes a weak thermoacoustic coupling, which is insufficient for stimulating any acoustic mode of the combustor. Although combustors are generally lightly damped, the acoustic damping of the combustor during the stable combustion is high enough to instantaneously subdue the eigenfrequency component emerging from the Gaussian background noise. Thermoacoustic interaction, which would be strong enough to excite an acoustic eigenmode of the combustor, triggers the transition of the stability of a combustion process. Once the instability associated with an acoustic mode of a combustor initiates, a sinusoid with the frequency of the eigenmode grows out of the background noise because the acoustic damping of the combustor would not be enough to suppress the growth of the sinusoid. The sinusoidal pattern of the eigenmode becomes more and more apparent in DP signal as time passes while the other components remain negligibly small. The amplitude of the sinusoidal wave nonlinearly increases as the thermoacoustic instability progresses and finally saturates at the limit cycle level.

TK mirrors the variation of the temporal characteristic of the DP signal. For DP signals in stable combustion, typically composed of Gaussian random noise and intermittent impulsive events, TK hovers around 3.0 or higher. A latent sinusoid of eigenfrequency exercises little influence on TK. Intermittent impulsive spikes generally increase the kurtosis of a non-impulsive signal. Once the excitation of an acoustic eigenmode of a combustion chamber begins, sinusoidal pattern emerges from the low level Gaussian noise. The sinusoidal pattern, a graphical indicator of transition of the combustion stability, drops TK below 3.0 because it adds the bimodal feature to the unimodal distribution of Gaussian noise [19]. The amplitude of the sinusoidal oscillation enlarges nonlinearly while the combustion process approaches the stability limit. The amplification of the sinusoid strengthens the bimodal characteristic of the signal and, as a result, TK declines toward 1.5 (i.e., the theoretical kurtosis value of a pure sine wave). For such a Gaussian random process with admixed sinusoid, the statistical distribution of the signal is mathematically described as a convolution of the two probability density functions (PDFs) [20]. If the combustion process enters the highly unstable regime, DP signal oscillates with the limit cycle amplitude. The limit cycle oscillation leads to the stabilization of TK slightly above 1.5. The stabilized TK value is dependent on the case-specific level of background noise. TK remains near the same level while the limit cycle oscillation continues.

In the previous paragraph, the variation of TK of a given DP signal is explained with regard to the progress of single-mode CI. The interpretation can be readily generalized for multi-mode CI with a little effort.

3. Feasibility of TK as a measure of CI

3.1. Fuel-rich combustion test with a rocket engine

The time trace of DP obtained from a fuel-rich combustion test with a rocket engine at around the onset of CI [21] is depicted in Fig. 1(a), which includes the transition from the stable to the unstable combustion. The sampling frequency of the data is 50 kHz. The zoomed-in plots of Fig. 1(b) to (d) represent the time histories of DP at three different phases of the combustion process. At the initial stage (i.e., 0–0.06 s), as shown in Fig. 1(b), the DP signal is nearly random. The randomness of DP signal weakens during the intermediate stage (i.e., 0.06–0.14 s) in Fig. 1(c). A sinusoidal wave competing with Gaussian random noise becomes more prominent within the random signal as time passes. The growth of the sinusoidal oscillation implies that an acoustic eigenmode of the rocket engine, which is associated with the eigenfrequency of 330 Hz in this example, is excited. In the final stage (i.e., 0.14–0.2 s), as depicted in Fig. 1(d), the dominant 330 Hz sine wave with gradually-increasing amplitude carries components of small ripples of background noise.

Histograms juxtaposed with the enlarged plots of Fig. 1(b) to (d) show the distribution of DP signal at each corresponding combustion process. The histograms of Fig. 1(b) and (c) commonly exhibit unimodal distribution with highest population of signal values at the bin near the mean pressure. However, the qualitative difference in signal characteristics of the two distributions other than the peak sharpness and the tail thickness is hardly discernable. The bimodal distribution of the histogram