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Research Paper

Absolute maximum heat transfer rendered by straight fins with quarter circle profile using Finite Element Analysis

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HIGHLIGHTS

- Straight fins with quarter circle profile are analyzed.
- The Biot number is the controlling parameter.
- The 2-D heat conduction equation is solved with Finite Element Analysis.
- Absolute and relative maximum heat transfer are compared.

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ABSTRACT

The aim of this paper is to assess succinctly the maximum heat transfer attributes of straight optimal fins with quarter circle profile proposed by Isachenko et al. in terms of three describable parameters. There are two thermal parameters: the thermal conductivity k and the mean convection coefficient $\bar{h}$, and one geometric parameter: the semi-thickness at the base R being equal to the length R. The peculiarity of this fin is its unitary slenderness ratio S (length $R \div$ semi-thickness at the base R), which contrasts markedly with all standard straight fins where the length L is always much larger than the thickness δ. Through an intuitive approach, Isachenko et al. called the straight fin with a quarter circle profile fin as the optimal straight fin capable of transferring maximum heat. In order to verify this premise qualitatively, detailed mean temperature distributions were obtained solving numerically the governing two-dimensional heat conduction equation in Cartesian coordinates with Finite Element Analysis. This was done within the spectrum of representative values of the radius-based Biot number, Bi. Additionally, a comparison was carried out between the maximum heat transfer produced by the straight fin with quarter circle conceived by Isachenko et al. and the longitudinal fin of concave parabolic profile envisaged by Schmidt to decide which of the two fin profiles renders absolute maximum heat transfer or relative maximum heat transfer.

1. Introduction

When a fin is attached to the surface of a heated solid body, the main purpose is to transfer heat by axial conduction in the fin and by transverse convection from the fin to the surrounding fluid where the fin is immersed. The primary objective of a fin is to enhance convective heat transfer by increasing the effective heat transfer area of the heated solid body. In general, there are two major fin arrangements as classified by Kraus et al. [1]: (1) straight or longitudinal fins and (2) annular or circumferential fins. In each arrangement, there are diverse types depending on the defining profile.

Typically, straight (or longitudinal) fins and annular (or circumferential) fins are much longer than they are thick. Hence, a traditional approximation used in fin analyses is to assume that (1) the temperature varies predominantly in the axial direction of the fin and (2) the transversal temperature gradients in the cross-section of the fin are virtually negligible. Because of these two features, it is common, and fairly accurate, to assume that the temperature variation in straight or annular fins occurs along the length only (Kraus et al. [1]). What results from this assumption is the quasi one-dimensional heat conduction analysis that appears in heat transfer textbooks.

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Nomenclature

A_{cb}	cross-sectional area at the base [m^2]
A_p	profile area [m^2]
Bi	Biot number, $\frac{\bar{h}R}{k}$
Bi_t	transversal Biot number for straight fins of rectangular and concave parabolic profile, $\frac{\bar{h}t}{k}$
$\bar{h}$	mean convection coefficient [W/(m^2 K)]
k	thermal conductivity [W/(m K)]
L	length for straight fins of rectangular and concave parabolic profile [L]
n	normal direction to the quarter circle
Q	heat transfer [W]
Q_{max}	maximum heat transfer [W]
R	length and semi-thickness at the base [L]
R_K	heat conduction resistance [K/W]
R_C	heat convection resistance [K/W]
S	slenderness ratio, length R ÷ semi-thickness at the base R
t	semi-thickness of straight fin of rectangular profile and semi-thickness at the base of straight fin of concave parabolic profile [L]
T	temperature [K]
T_f	free-stream temperature [K]
$T - T_f$	excess temperature [K]
W	depth [L]
x	axial (horizontal) coordinate [L]
X	dimensionless axial coordinate x, $\frac{x}{R}$
y	transversal (vertical) coordinate [L]
Y	dimensionless transversal coordinate y, $\frac{y}{R}$

Greek symbols

δ	thickness of straight fin of rectangular profile and thickness at the base of straight fin of concave parabolic profile [L]
θ	dimensionless excess temperature $T - T_f$, $\frac{T-T_f}{T_b-T_f}$

Subscripts

b	base
c	mid-plane
m	mean
opt	optimal
s	surface
t	tip

According to heat conduction theory (Arpaci [2]), the rigorous analysis of straight and annular fins requires the use of an accurate two-dimensional model, rather than the approximate quasi one-dimensional model. To the authors' knowledge, the two-dimensional treatment of straight fins has been mostly done for the trapezoidal profile with straight sides (wherein the rectangular profile and triangular profile are particular cases). Representative references are cited in chronological order: Lau and Tan [3], Chung et al. [4], Look [5], Aparecido and Cotta [6], Huang and Shah [7], Razelos and Georgiou [8], Juca and Prata [9], Aziz [10], Yeh [11], Onur [12], Kang and Look [13], Casarosa and Franco [14], Singh et al. [15], Xia and Jacobi [16], Marin et al. [17], Kang [18], Gorobetz [19], Malik and Rafiq [20], Karvinen and Karvinen [21], Kundu et al. [22], Lindstedt et al. [23], Iqbal et al. [24], Nguyen and Yang [25], and Fyrillas and Leontiou [26]. Incidentally, the other type of straight fins, called pin fins or spines have been excluded in the list.

Contrarily, a literature review demonstrated that most straight fins of variable profile (with non-straight sides) have been considered in the past within the simplified framework of quasi one-dimensional heat conduction (Kraus et al. [1]), and not within the realistic framework of two-dimensional heat conduction.

Shifting the attention to optimization theory, straight fins of optimal profile are defined as those fins that maximize heat transfer per unit volume of material. In reference to this, there are two schools of thought. In one school of thought, Schmidt [27] adduced that for a straight fin of variable profile to be optimal, the temperature gradient would be constant everywhere and, as a by-product the heat flux was uniform over the fin. This pattern indicated that the temperature would be, in fact, a linear function of the axial coordinate connoting that the heat flow was one-dimensional. In this sense, Schmidt [27] advocated that the optimum profile of the straight fin has to be concave parabolic. Years later, Duffin [28] published a rigorous proof of the optimality entrusted to the straight fin of concave parabolic profile employing calculus of variations. In the other school of thought, Isachenko et al. [29] hypothesized that for a straight fin to deliver maximum heat transfer, the heat flux has to be constant in all transverse sections of the fin, i.e., the heat flow lines have to be parallel to the axial coordinate x. As a consequence of the heat flux behavior, the axial temperature gradient is uniform, and therefore the temperature varies linearly in the x-direction along the fin. Afterwards, these authors concluded that for the straight fin with variable profile to be optimal and deliver maximum heat transfer, the fin profile should be formed with the intersection of two equal circles with radius R.

The first objective of the technical paper is to scrutinize mathematically whether the straight fin with quarter circle proposed by Isachenko et al. [29] has indeed an optimal shape and delivers maximum heat transfer. Essentially, the present study constitutes the counterpart of Duffin's work. In this regard, the irregular-shaped fin in question will be modeled as a two-dimensional region and the temperatures will be determined by Finite Element Analysis. The second objective deals with the one-to-one comparison between the maximum heat transfer produced by the straight fin with quarter circle conceived by Isachenko et al. [29] and the straight fin of concave parabolic profile envisaged by Schmidt [27] to decide which of the two fin profiles renders absolute maximum heat transfer.

The present technical paper is organized as follows. The straight fin with quarter circle profile as perceived by Isachenko et al. [29] is described in Section 2. The pertinent physical system and the mathematical formulation are illustrated in Section 3. Section 4 consists in the Finite Element Analysis, the computational domain and the adequate mesh. The presentation and discussion of results are disclosed in Section 5 for representative values of the controlling parameter, the Biot number. In addition, Section 5.4 elucidates two illustrative examples to compare and rank the maximum heat transfer associated with the two optimal straight fins expounded by Isachenko et al. [29] and Schmidt [27].

2. Straight fin with quarter circle profile

Consider a family of straight fins of variable profile $y(x)$ and large depth W attached at a heated solid body in Fig. 1. The fin base is at a high temperature T_b and loses heat to a fluid flowing across its upper and lower surfaces at a low free-stream temperature T_f. It is presumed that the thermal conductivity k of the solid is constant and the mean convection coefficient $\bar{h}$ at the solid/fluid interface is uniform. From heat conduction theory (Arpaci [2]), the formulation