Journal of Applied Geophysics 132 (2016) 33–37

Contents lists available at ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

Singular spectral analysis based filtering of seismic signal using new Weighted Eigen Spectrogram

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ARTICLE INFO

Article history:
Received 5 April 2015
Accepted 28 June 2016
Available online 30 June 2016

Keywords:
Singular spectral analysis
Frequency filtering
Eigenvectors
Seismic reflection data

ABSTRACT

Filtering of non-stationary noisy seismic signals using the fixed basis functions (sine and cosine) generates artifacts in the final output and thereby leads to wrong interpretation. In order to circumvent the problem, we propose here, a new Weighted Eigen Spectrogram (WES) based robust time domain Singular Spectrum Analysis (SSA) frequency filtering algorithm. The new WES is used to simplify the Eigen triplet grouping procedure in SSA. We tested the robustness of the algorithm on synthetic seismic data assorted with field-simulated noise. Then we applied the method to filter the high-resolution seismic reflection field data. The band pass filtering of noisy seismic records suggests that the underlying algorithm is efficient for improving the signal to noise ratio (S/N) and also it is user-friendly.

1. Introduction

Frequency domain filtering is a very useful tool for analyzing signal and noise from of field seismic reflection data. Researchers have employed several filtering techniques to filter the unwanted signal from the band limited seismic reflection data (Yilmaz, 1987; Yilmaz, 2001) using fixed sine and cosine basis functions. These techniques rely on the assumptions of stationary behavior of the data. However, in practice most of seismic field data show spatio-temporal non-stationary and non-linear behaviors. Consequently, application of the above filtering techniques for non-stationary signal produces artifacts in the final filtered output (Rajesh et al., 2014). Ghil and Taricco (1997) have suggested Singular Spectrum Analysis (SSA) as a robust method to decompose the non-stationary data with sudden changes, jumps, and gaps using minimum number of Eigen modes (Rajesh et al., 2014). Recently, Bozzo et al. (2010) have reported the analogy between Fourier spectral components and Eigen modes of the SSA. Several other researchers have independently given the spectral interpretation of the independent Eigen/principal modes of SSA (Harris and Yan, 2010; Golyandina and Zhigljavsky, 2013). In a recent work, Rajesh et al. (2014) have demonstrated the application of SSA based time domain frequency filtering technique for seismic reflection data to overcome the problem of artifacts. The comparative study with SSA based frequency-filtering operation using the Fourier and f–x SSA methods has shown that the artifacts arising due to domain conversion are not present in the t–x SSA filtering scheme (Rajesh et al., 2014; Rajesh and Tiwari, 2015). Hence, the SSA based filtering would be the appropriate choice to analyze such data sets for better signal reconstruction (Yiou et al., 2000; Rajesh et al., 2014).

However, the identification of periodicities from the Eigenvectors, as proposed in our earlier work is not user friendly for its robust and wider applicability. Hence, we develop here a user-friendly WES based algorithm for wider adaptability of SSA frequency filtering (Rajesh et al., 2014; Rajesh and Tiwari, 2015). Hence, the purpose of this paper is to (i) demonstrate the new Weighted Eigen Spectrogram based robust and user-friendly SSA frequency filtering algorithm (ii) test the proposed algorithm on synthetic data and (iii) demonstrate its application to high-resolution seismic reflection field data.

2. Methodology

The singular spectral analysis (SSA) is a robust tool to identify the periodic components of a time series immersed in noise (Broomhead and King, 1986a, 1986b; Fraedrich, 1986; Vautard and Ghil, 1989; Vautard et al., 1992; Golyandina et al., 2001; Ghil et al., 2002; Oropeza and Sacchi, 2011, Rajesh et al., 2014). Here we used the methodology of SSA to filter the each trace of 2D seismic data given by D (t, x)

$$\mathbf{D}\,(\mathbf{t},\mathbf{x})_{(N_t\times N_x)} = \begin{bmatrix} \mathbf{s}(1,1) & \mathbf{s}(1,2) & \cdots & \mathbf{s}(1,\mathbf{N_x}) \\ \vdots & & \ddots & \vdots \\ \mathbf{s}(\mathbf{N_t},1) & \mathbf{s}(\mathbf{N_t},2) & \cdots & \mathbf{s}(\mathbf{N_t},\mathbf{N_x}) \end{bmatrix} \quad (1)$$

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http://dx.doi.org/10.1016/j.jappgeo.2016.06.009
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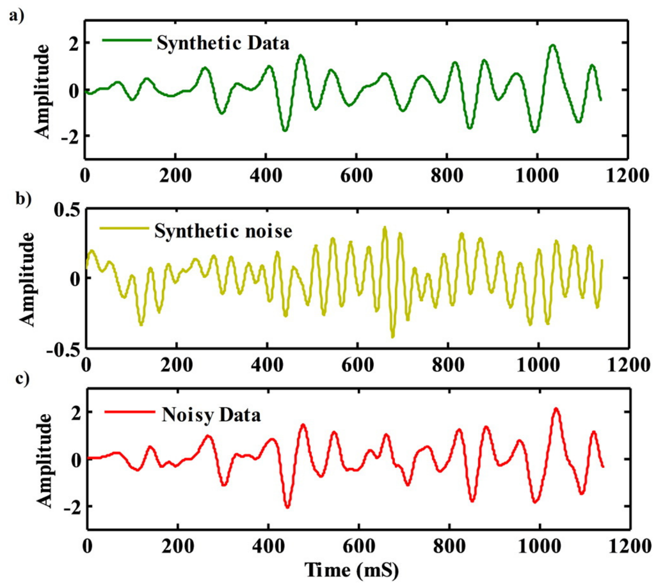

Fig. 1. Synthetic data filtering (a) synthetic trace (b) noise (c) synthetic data added with noise.

The elements **s (i, j)** represents the reflection signal amplitudes corresponds to time i ($0 < i \le N_t$) and of jth($0 < j \le N_x$) trace. Here N_t and N_x respectively represent the number of sampling times and number of receiver channels.

(i) Trajectory matrix formulation: To implement the SSA algorithm on the ith trace represented by $S(t,i) = \{s(1,\ i)\ s(2,\ i) \ldots\ldots\ s(\ N_t, i)\}$, we formulate the trajectory/Hankel matrix of the trace using an appropriate window length (L) as shown below.

$$T_{i(L \times K)} = \begin{bmatrix} s(1,i) & s(2,i) & \cdots & s(K,\ i) \\ s(2,i) & s(3,i) & \cdots & s(K+1,i) \\ \vdots & \ddots & & \vdots \\ & & \ddots & \\ \vdots & & & \vdots \\ s(\ L,i) & & \cdots & s(N_t, i\ ) \end{bmatrix} \quad (2)$$

Here $K = N - L + 1$ and $0 < i \le N_x$. The selection of optimal window (L) is crucial in SSA frequency filtering algorithm. In general, the window length should be at least equal to the highest period of the low frequency component that is present in the data within the classical limit $2 < L \le N_t / 2$. In the present analysis, we have chosen the window length as more than double the period corresponding to the lowest frequency of our interest. Using the Hankel operator O_h, we can write the equation 2 as follows

$$T_{i(L_x \times K_x)} = O_h \cdot S(t,i) \quad (3)$$

(ii) Singular Value Decomposition of Trajectory matrix: The next stage of the algorithm is the Singular Value Decomposition (SVD) of the trajectory matrix given by

$$T_i = \sum_{i=1}^{d} \sqrt{\lambda_i}\, U_i.V_i^T \quad (4)$$

Here U_i, V_i are the singular vectors corresponds to the ith non-zero singular value $\sqrt{\lambda_i}$, V_i^T represents the transpose of V_i and d is the number of Eigen components with non-zero Eigen value. The group ($\sqrt{\lambda_i}$, U_i, V_i) is called the ith Eigen triplet and $\lambda_1 > \lambda_i > 0$ for i = 1, 2… d.

In essence, through the decomposition operation denoted by O_d we will obtain Eigenvectors, Eigen values of the trajectory matrix as follows

$$\text{i.e.,}\quad T_i = \sum_{i=1}^{d} \sqrt{\lambda_i}\, U_i.V_i^T = O_d.T_i = O_d.O_h.S(\ t, i\ ) \quad (5)$$

(iii) Grouping from Weighted Eigen Spectrogram:

Now we will construct a matrix $E_w = [\sqrt{\lambda_1}U_1 : \sqrt{\lambda_2}U_2 : \ldots.. : \sqrt{\lambda_k}U_k]$ of K Weighted Eigenvectors. We apply the Fourier transform on E_w to convert them into frequency domain. Even though we use the Fourier transform to visualize the spectral content, the decomposition and the reconstruction of the signal in the filtering operation is purely in the time domain and uses the data adaptive Eigenvectors. Finally, we compute the power spectral estimates of each Weighted Eigenvector to generate the WES, a contour of power on the frequency - Eigen triplet number 2D plot as shown in Fig. 2.

(iv) Reconstruction: In this stage, we reconstruct the trajectory matrix (T^r) from '**k**' Eigen triplets selected from the WES.

$$T^r = \sum_{i=1}^{k} \sqrt{\lambda_i}U_i.V_i^T \quad (6)$$

We denote the above operation using reconstruction operator O_r and then using the Eqs. (5) and (6), T^r can be written as follows

$$\text{i.e.,}\quad T^r = O_r.T_d = O_r \cdot O_d \cdot O_h \cdot S(i, x) \quad (7)$$

In the final step, we average the anti-diagonal elements of the reconstructed trajectory matrix T^r to obtain the filtered data series $S_r\ (t,\ i)$ where $1 \le i \le N_x$. The computation of the elements of S_r is as follows.

$$S_r\ (k, i) = \frac{1}{k}\sum_{m=1}^{k} x_{m,\ k-m+1} \quad \text{for } 0<k<L \quad (8a)$$

$$S_r\ (k, i) = \frac{1}{L}\sum_{m=1}^{L} x_{m,\ k-m+1} \quad \text{for } L-1<k<K+1 \quad (8b)$$

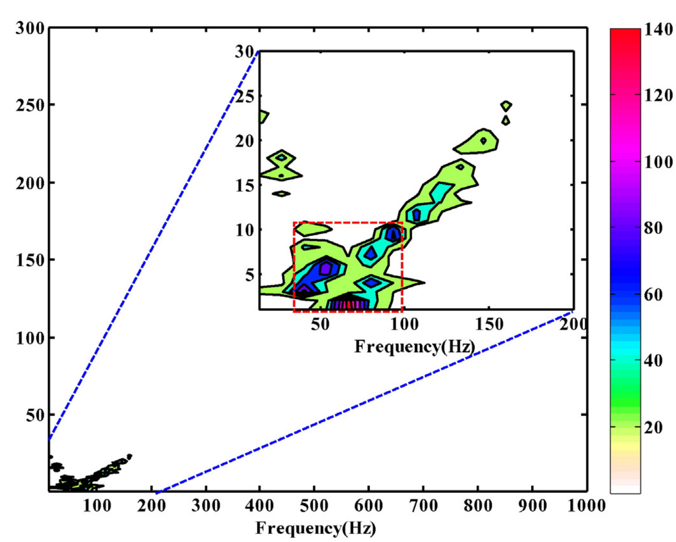

Fig. 2. Weighted Eigen Spectrogram of used for grouping the Eigen triplets corresponding to frequency 30 to 100 Hz and enlarged image of WES showing (in red color rectangular box) the spectral power at these frequencies. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)