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Improved free-surface expression for frequency-domain elastic optimal mixed-grid modeling

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ABSTRACT

An accurate and efficient forward modeling is the foundation of full-waveform inversion (FWI). In elastic wave modeling, one of the key problems is how to deal with the free-surface boundary condition appropriately. For the representation of the free-surface boundary condition, conventional displacement-based approaches and staggered-grid approaches are often used in time-domain. In frequency-domain, considering the saving of storage and CPU time, we integrate the idea of physical parameter-modified staggered-grid approach in time-domain with an elastic optimal mixed-grid modeling scheme to design an improved parameter-modified free-surface expression. Accuracy analysis shows that an elastic optimal mixed-grid modeling scheme using the parameter-modified free-surface expression can provide more accurate solutions with only 4 grid points per smallest shear wavelength than conventional displacement-based approaches and is stable for most Poisson ratios. Besides, it also yields smaller condition number of the resulting impedance matrix than conventional displacement-based approaches in laterally varying complex media. These advantages reveal great potential of this free-surface expression in big-data practical application.

1. Introduction

Full-waveform inversion (FWI) is an attracting method to offer the possibility of revealing underground detailed structure and reservoir properties in oil and gas exploration. It is a data-fitting process based on full-waveform modeling to extract physical properties from seismic data (Operto and Virieux, 2009). The elastic wave equation is an approximate modeling tool to simulate the propagation of seismic waves in complex media. Modeling in frequency-domain has the advantages of high efficiency for multi-shot computation using a direct solver, single frequency manipulation for inversion, and flexible simulation of attenuation effect (Pratt, 1990; Jo et al., 1996).

A major obstacle in the frequency-domain modeling is the storage requirement and solving an extremely large matrix equation. Minimizing the size of the matrix will be an effective way to overcome this problem. Based on a rotated operator, Jo et al. (1996) designed an optimal mixed-grid operator for 2-D scalar wave modeling. It reduced the number of grid points per smallest wavelength to approximately 4, leading to a significant reduction of matrix size at the cost of increasing the numerical bandwidth of matrix. Štekl and Pratt (1998) extended this method to 2-D elastic wave modeling and reduced the number of grid points per smallest shear wavelength to approximately 4 as well. The difference is that it uses the same grid points in the numerical computation as an original second-order difference scheme (Kelly et al., 1976), thus extra numerical bandwidth is not needed. However, for elastic modeling, the elastic free-surface boundary condition is a key issue requiring special considerations. In time-domain, there are two kinds of method to deal with the free-surface boundary condition proposed in the past few decades. One is the displacement-based approach, which uses the approximate spatial derivative of the displacement to express the free-surface boundary condition, including the centered explicit approximation (Alterman and Karal, 1968), the one-side explicit approximation (Alterman and Rotenberg, 1969; Kelly et al., 1976), the centered implicit approximation (Vidale and Clayton, 1986), the composed approximation and some of its revisions (Ilan et al., 1975; Ilan and Loewenthal, 1976; Lan and Zhang, 2011). For frequency-domain implementation, composed approximations are complicated and inapplicable, because they use x and t derivatives to replace the z derivatives on the free-surface, involving more

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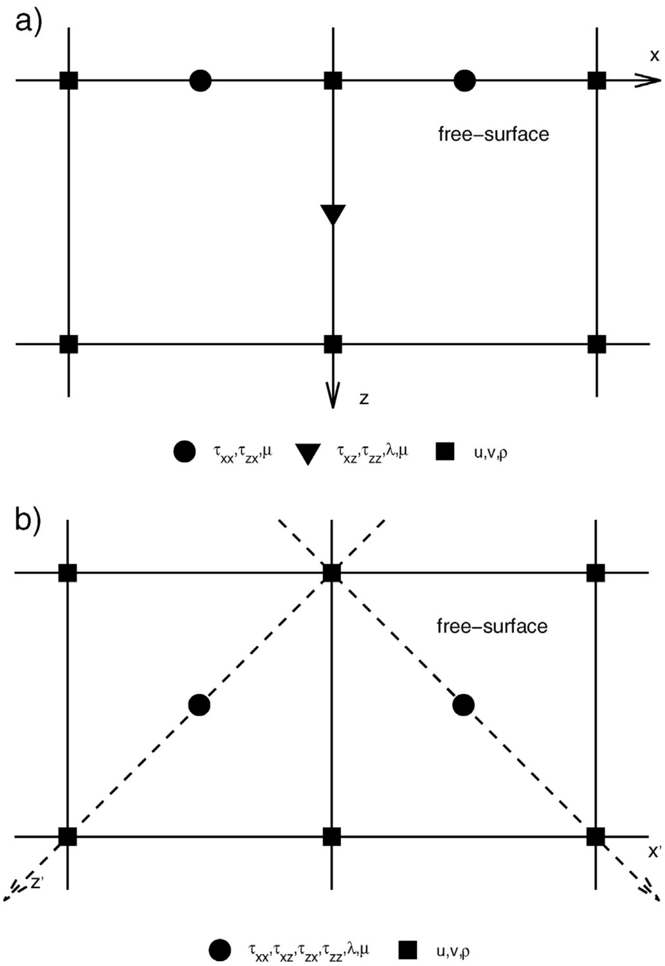

Fig. 1. Staggered-grid geometries for the free-surface. (a) Cartesian system staggered-grid scheme. (b) Rotated system staggered-grid scheme.

calculations than the others. The other is the staggered-grid approach based on the Virieux's explicit numerical scheme of velocity-stress elastodynamics equation system (Virieux, 1986). For instance, vacuum formalism (Zahradník et al., 1993) sets the Lamé parameters above the free-surface to zero and the density close to zero to implicitly approximate the free-surface. The stress image method (Levander, 1988; Graves, 1996; Gottschämer and Olsen, 2001) images stress fields across the free-surface as the odd functions. The physical parameter-modified methods, which describe the stress-free by a modification of physical parameters on the free-surface without extra processing above the free-surface, are simple and easy to apply to frequency-domain modeling. Mittet (2002) used a transversely isotropic approach to the free-surface. Xu et al. (2007) pointed out that Mittet's scheme violates the basic physical principle, which addresses dispersion in terms of their numerical examples (Bohlen and Saenger, 2006; Xu et al., 2007). They implemented the free-surface condition by an acoustic-elastic boundary approach.

Our work is based on the 2-D elastic wave equation in frequency-domain described by displacement and stress. Two numerical operators of Štekl and Pratt (1998), a differencing operator in a rotated coordinate system and a lumped mass term, are used to discrete the displacement-stress equation system in the same way as the elastic optimal mixed-grid modeling scheme. For the grid points near the free-surface, a parameter-modified staggered-grid method is used in both Cartesian and rotated coordinate system difference schemes, respectively, which gives an improved parameter-modified free-surface expression. The numerical examples show that: (1) for Lamb's problem, an elastic optimal mixed-grid modeling scheme using the parameter-modified free-surface expression can provide more accurate solutions with only 4 grid points per smallest shear wavelength than conventional displacement-based approaches and is stable for most Poisson ratios; (2) for laterally varying complex media modeling, it also yields better solution and smaller condition number than conventional displacement-based approaches.

2. Elastic modeling for free-surface

Elastic modeling in frequency-domain is often implemented on the displacement-based elastic wave equation,

$$\begin{cases} \omega^2\rho u + \dfrac{\partial}{\partial x}\left[\lambda\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial z}\right)+2\mu\dfrac{\partial u}{\partial x}\right]+\dfrac{\partial}{\partial z}\left[\mu\left(\dfrac{\partial u}{\partial z}+\dfrac{\partial v}{\partial x}\right)\right]+f=0, \\ \omega^2\rho v + \dfrac{\partial}{\partial z}\left[\lambda\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial z}\right)+2\mu\dfrac{\partial v}{\partial z}\right]+\dfrac{\partial}{\partial x}\left[\mu\left(\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial z}\right)\right]+g=0, \end{cases} \tag{1}$$

where ω is the angular frequency, f and g are the horizontal and vertical body forces, u and v are the horizontal and vertical components of Fourier transformed displacements, respectively, λ and μ are the Lamé parameters, and ρ is the density. Štekl and Pratt (1998) introduced two separate coordinate systems, one rotated 45° with respect to the other, and a lumped mass term to present an elastic optimal mixed-grid modeling scheme for Eq. (1). It reduces the number of grid points per smallest shear wavelength to approximately 4 and works well for the interior part of the model.

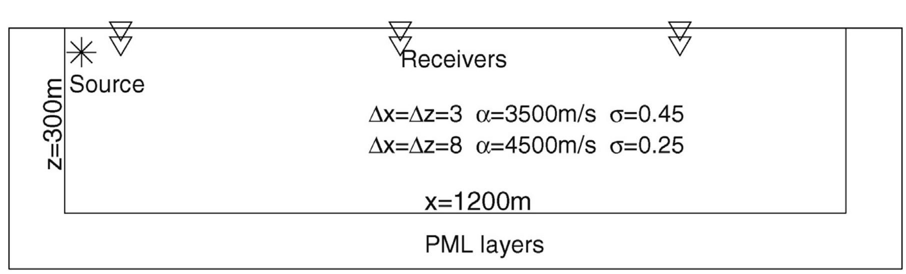

Fig. 2. Acquisition schematic of the homogenous model.