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Adaptive finite element modeling of direct current resistivity in 2-D generally anisotropic structures

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ABSTRACT

In this paper, we present an adaptive finite element (FE) algorithm for direct current (DC) resistivity modeling in 2-D generally anisotropic conductivity structures. Our algorithm is implemented on an unstructured triangular mesh that readily accommodates complex structures such as topography and dipping layers and so on. We implement a self-adaptive, goal-oriented grid refinement algorithm in which the finite element analysis is performed on a sequence of refined grids. The grid refinement process is guided by an a posteriori error estimator. The problem is formulated in terms of total potentials where mixed boundary conditions are incorporated. This type of boundary condition is superior to the Dirichlet type of conditions and improves numerical accuracy considerably according to model calculations. We have verified the adaptive finite element algorithm using a two-layered earth with azimuthal anisotropy. The FE algorithm with incorporation of mixed boundary conditions achieves high accuracy. The relative error between the numerical and analytical solutions is less than 1% except in the vicinity of the current source location, where the relative error is up to 2.4%. A 2-D anisotropic model is used to demonstrate the effects of anisotropy upon the apparent resistivity in DC soundings.

1. Introduction

It is commonly known that the layered sedimentary sequences can exhibit macroscopic electrical anisotropy with a symmetry axis perpendicular to the bed plane, and fracture zones often show significantly orientational preferences giving rise to a more macroscopic electrical anisotropy (Keller and Frischknecht, 1996; Bhattacharya and Patra, 1968; Li and Spitzer, 2005). Thus, ignoring electrical anisotropy when dealing with the field data acquired in areas of layered sedimentary rocks and fractured zones may lead to a distorted image of conductivity structures, even misinterpretation (Asten, 1974; Li and Dai, 2011).

The electrical potential due to a point source in a homogeneous anisotropic medium can be obtained by transforming Laplace's equation for a homogeneous isotropic medium into Laplace's equation for a homogeneous anisotropic medium by stretching and rotating a coordinate system (Bhattacharya and Patra, 1968; Habberjam, 1979).The quasi-analytical solution in arbitrarily anisotropic layered media and the effects of anisotropy for a layered structure upon the direct current resistivity was investigated by Wait (1990), Li and Uren (1997) and Pervago et al. (2006). For a general anisotropic earth, 3-D direct current resistivity finite element modeling algorithms were developed by using structured grids (Pain et al., 2003; Li and Spitzer, 2005), unstructured grids (Wang et al., 2013) and the Gaussian quadrature grids (Zhou et al., 2009). Verner and Pek (1998) presented a 2-D finite difference (FD) method for numerical modeling of DC resistivity in anisotropic structures, in which the partial differential equations are discretized on a rectangular cell and the Dirichlet boundary condition is applied. Bibby (1978) presented a finite element code for axially symmetric bodies, which allows for some simple cases of anisotropy. Zhou et al. (2009) reported a direct current resistivity modeling method in dipping anisotropic media using a Gaussian quadrature grid.

We sought to develop a new numerical method that can handle complex models including rough topography and simulate geoelectrical responses in arbitrarily anisotropic structures. Furthermore, we wanted to improve the numerical precision with the incorporation of the mixed boundary condition and the use of the adaptive mesh refinement scheme.

In this paper, we develop a new finite element resistivity modeling method in a general 2-D anisotropic conductivity structure. First, we describe the numerical realization of the adaptive finite element algorithm in details. Then, we verify the finite element algorithm and code using an anisotropic two-layered model. Finally, we use 2-D anisotropic models to demonstrate the effects of anisotropy upon the direct current resistivity.

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2. Methods

2.1. Governing equations and boundary conditions

Consider a 2-D conductivity model with general anisotropy and let x be the structural strike direction. The coordinate system is right-handed with the z-axis pointing positive downwards. In the anisotropic earth, the current density $\mathbf{j}$ and the electric field $\mathbf{E}$ are in general no longer parallel each other, which is expressed by the generalized Ohm's law

$$\mathbf{j} = \underline{\underline{\sigma}}\mathbf{E}, \tag{1}$$

where $\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$ is the conductivity tensor. In the earth, it is symmetric and positive definite.

The conductivity tensor $\underline{\underline{\sigma}}$ can be rotated into its principal axes (x', y', z') and described by its three principal conductivities $\sigma_{x'}, \sigma_{y'}, \sigma_{z'}$ and the corresponding three Euler angles α_s (anisotropy strike), α_d (anisotropy dip), α_l (anisotropy slant). The transformation is carried out by successively applying three rotation matrices to the conductivity tensor corresponding to the Euler angles: first around the z-axis by α_s, then around the new x-axis by α_d, and finally around the new z-axis by α_l.

$$\underline{\underline{\sigma}} = R_z(-\alpha_s)R_x(-\alpha_d)R_z(-\alpha_l)\begin{pmatrix} \sigma_{x'} & 0 & 0 \\ 0 & \sigma_{y'} & 0 \\ 0 & 0 & \sigma_{z'} \end{pmatrix}R_z(\alpha_l)R_x(\alpha_d)R_z(\alpha_s),$$

where $R_z(\alpha_s)$, $R_z(\alpha_d)$ and $R_z(\alpha_l)$ are the rotation matrices.

Assume that the steady electric field is generated by a current source I, which is located at a point (x_q, y_q, z_q) in the conductive earth, then, we have

$$\nabla \cdot \mathbf{j}(x,y,z) = I\delta(x-x_q)\delta\left(y-y_q\right)\delta(z-z_q), \tag{2}$$

where δ is the Dirac delta function and $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$.

The electrical field $\mathbf{E}(x,y,z)$ can be represented by the negative gradient of the electric potential $v(x,y,z)$

$$\mathbf{E}(x,y,z) = -\nabla v(x,y,z). \tag{3}$$

Substituting Eqs. (1) and (3) into Eq. (2), yields

$$\nabla \cdot \left(\underline{\underline{\sigma}}\nabla v(x,y,z)\right) = -I\delta(x-x_q)\delta\left(y-y_q\right)\delta(z-z_q). \tag{4}$$

Although the conductivity distribution is 2-D, Eq. (4) is a 3-D differential equation. It can be transferred into a 2-D equation by using Fourier transformation with respect to the strike direction x

$$V(k,y,z) = \int_{-\infty}^{\infty} v(x,y,z)e^{-ikx}dx, \tag{5}$$

resulting in the following partial differential equation

$$\nabla_2 \cdot \left(\underline{\underline{\tau}}\nabla_2 V\right) - \sigma_{xx}k^2 V + ik\nabla_2 \cdot \mathbf{p} + ik\left(\sigma_{xy}\frac{\partial V}{\partial y} + \sigma_{xz}\frac{\partial V}{\partial z}\right) = -I\delta\left(y-y_q\right)(z-z_q)e^{-ikx_q}, \tag{6}$$

with $\nabla_2 = (\frac{\partial}{\partial y}, \frac{\partial}{\partial z})$, $\underline{\underline{\tau}} = \begin{pmatrix} \sigma_{yy} & \sigma_{yz} \\ \sigma_{zy} & \sigma_{zz} \end{pmatrix}$, $\mathbf{p} = \sigma_{xy}V\mathbf{n}_y + \sigma_{xz}V\mathbf{n}_z$, where k is the wave number, and $\mathbf{n}_y$ and $\mathbf{n}_z$ are the unit vectors along the y and z axis, respectively.

In order to solve Eq. (6), the boundary conditions must be applied. On the interface between two conductivity bodies with different conductivity tensor denoted by Γ_I, the electric potential v must be continuous, thus we have

$$v_1(x,y,z) = v_2(x,y,z) \quad \text{on} \quad \Gamma_I \tag{7}$$

in the space domain and

$$V_1(k,y,z) = V_2(k,y,z) \quad \text{on} \quad \Gamma_I \tag{8}$$

in the wave number domain.

With the use of Eqs. (5) and (3), the generalized Ohm's law (Eq. (1)) in the space domain can be transformed into

$$\mathbf{J}(k,y,z) = -ik\sigma_{xx}V\mathbf{n}_x - ik\mathbf{p} - \left(\sigma_{xy}\frac{\partial V}{\partial y} + \sigma_{xz}\frac{\partial V}{\partial z}\right)\mathbf{n}_x - \underline{\underline{\tau}}\nabla_2 V \tag{9}$$

in the wave number domain, where $\mathbf{n}_x$ is the unit vector along the x axis.

Since there is no current flow through the air–earth interface denoted by Γ_s and the normal component of the current density is continuous at the interface, thus we have

$$\mathbf{J}(k,y,z) \cdot \mathbf{n} = 0 \quad \text{on} \quad \Gamma_s \tag{10}$$

where $\mathbf{n}$ is the unit normal vector of the boundary interface.

Substituting Eq. (9) into (10), yields

$$ik\mathbf{p} \cdot \mathbf{n} + \underline{\underline{\tau}}\frac{\partial V}{\partial n} = 0 \quad \text{on} \quad \Gamma_s. \tag{11}$$

On the other domain boundaries denoted by Γ_∞, the mixed boundary conditions are applied. Dey and Morrison (1979) proposed mixed boundary conditions for an isotropic medium. In an anisotropic 3-D case, such mixed boundary condition was derived by Li and Spitzer (2005).

In a generally anisotropic 2-D case, the mixed boundary conditions in the wave number domain can also be derived (see Appendix A for details).

$$\frac{\partial V}{\partial y} + \left(\rho_{yy}|y-y_q| + \rho_{yz}|z-z_q| - \rho_{xy}x_q - C\rho_{xy}\right)\beta_1 kV - ik\frac{\rho_{xy}}{\rho_{xx}}V = 0, \quad \in\Gamma_\infty \tag{12}$$

$$\frac{\partial V}{\partial z} + \left(\rho_{zz}|z-z_q| + \rho_{yz}|y-y_q| - \rho_{xz}x_q - C\rho_{xz}\right)\beta_1 kV - ik\frac{\rho_{xz}}{\rho_{xx}}V = 0, \quad \in\Gamma_\infty \tag{13}$$

with $C = \frac{\rho_{xy}|y-y_q| + \rho_{xz}|z-z_q| - \rho_{xx}x_q}{\rho_{xx}}$, $\beta_1 = \frac{K_1(k\sqrt{\frac{C_1}{\rho_{xx}}})}{K_0(k\sqrt{\frac{C_1}{\rho_{xx}}})}\frac{1}{\sqrt{\rho_{xx}C_1}}$, $K_0(k\sqrt{\frac{C_1}{\rho_{xx}}})$ and $K_1(k\sqrt{\frac{C_1}{\rho_{xx}}})$ are the modified Bessel functions of the second kind of order 0 and 1, respectively.

2.2. Finite element approximation

The method of weighted residuals is used to derive the FE approximation (Zienkiewicz, 1977; Li and Pek, 2008). Eq. (6) is multiplied by an arbitrary variation of the transformed electrical potential δV and integrated over the model area Ω, and, subsequently, modified by using