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Research Paper

Investigation of forced convection through entrance region of a porous-filled microchannel: An analytical study based on the scale analysis

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HIGHLIGHTS

- Performing the scale analysis for developing heat transfer inside porous materials;
- Analytical study of thermally developing forced convection in porous-microchannels;
- Effects of slip-flow regime on the Nusselt number and thermal entry length;
- Finding model B to yield inconsistent results with the LTE limit in slip-flow;
- Discussing on the thermally fully developed flow assumption in slip-flow regime.

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ABSTRACT

The thermally developing forced convection heat transfer in a micro-channel filled with a porous material in the slip-flow regime is analyzed. Channel walls are subjected to a constant heat flux. The local thermal non-equilibrium (LTNE) condition is considered and both the fluid and solid phases in the porous region are assumed to have internal heat generation. According to a perturbation analysis assuming small temperature difference between the two phases obtained by the scale analysis, we show that there is no need to apply a thermal boundary condition model at the channel wall. Thus, we obtained an analytical solution for the thermally developing Nusselt number (Nu) using no model. Thermal boundary condition models (A and B) are also used to find the temperature jump at the wall. Comparing Nu of models A and B with the pure perturbation analysis (using no model) and with the solution under local thermal equilibrium (LTE) condition reveals that model B cannot predict the LTE condition when a temperature jump exists on the wall. Hence, model A may be the only valid scenario in the slip-flow regime. In addition, expressions for the thermal entry length ($x_{developing}$) are proposed. An increase in β as well as a decrease in the thermal conductivity ratio (k) decrease $x_{developing}$.

1. Introduction

Miniaturizing the devices along with an ever-increasing demand for enhancing their performance opens a new field of study, the heat and fluid flow through micro-sized passages [1,2]. The increasing interests and requests concerning this field of study are evident by the number of recent articles published in the past few years [3,4]. On the other hand, increasing the heat transfer rate ability of a heat exchanger by inserting porous materials within the fluid flow passage has brought a relatively new idea of implementation of porous inserts within micro flow-passages pioneered by Haddad et al. [5] and Nield and Kuznetsov [6]. As recent analytical studies of forced convection heat transfer through porous-filled micro-passages in the slip-flow regime (i.e. Knudsen number, $Kn < 0.1$) the following can be mentioned: micro-porous rectangular ducts by Hooman [7], different thermal boundary condition models by Mahmoudi [8], porous micro-tubes and porous micro-annulus by Wang et al. [9,10], heat flux splitting (bifurcation) and conjugate heat transfer phenomena by Dehghan et al. [11], asymmetric heat flux by Xu et al. [12], nanofluid utilization by Ting et al. [13], and the heat transfer performance study (i.e. HTP, the ratio of the heat transfer rate enhancement to the pressure drop increment) by Dehghan et al. [14].

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Although flow is naturally a thermally developing phenomenon [15,16], only one analytical study by Kuznetsov and Nield [17] exists within the literature investigating the problem of developing heat transfer within porous materials in the slip-flow regime. In addition, it is worth mentioning that the thermally developing heat transfer analysis seems to be more crucial for micro-passages due to relatively small axial length compared to the conventional macro-fluid passages. Kuznetsov and Nield [17] applied an adaption of the Graetz methodology based on the local thermal equilibrium condition (LTE, i.e. the fluid and solid phases of the porous material have identical temperatures). The present study aims at investigating the thermally developing forced convective heat transfer inside microchannels filled with a porous material under the local thermal non-equilibrium (LTNE) condition, by presenting both analytical and numerical solutions. To cover a wider range and to present the solution of a more general classic problem, effects of the internal heat generation or absorption within the fluid and solid phases of the porous material are considered according to studies of Mahmoudi [8], Yang and Vafai [18] and Dehghan [19] that were performed for the thermally fully developed flow. In the present study, effects of pertinent parameters (k, Bi, β, and ω) on the Nusselt number, Nu, and the dimensionless thermal entry length, x_{dev}, are discussed.

2. Mathematical modeling

Fig. 1 represents a schematic diagram of the problem. An isotropic, homogenous, and rigid porous medium is considered in a 2D channel with impermeable parallel-plates. The channel has a height of $2H$ and a length of L. We investigate the thermally developing forced convective flow in the channel. The Darcy's law of motion in the laminar flow regime is studied. Steady flow with initial temperature of T_i enters the channel. The local thermal non-equilibrium (LTNE) condition is also considered between the two phases in the porous medium. Because of the symmetry, only a half of the channel is considered.

According to the above assumptions the energy conservation equations within the solid and fluid phases of the porous material are [16]:

$$\rho c_p u \frac{\partial T_f}{\partial x^*} = k_{f,eff} \frac{\partial^2 T_f}{\partial y^{*2}} + h_{sf} a_{sf} (T_s - T_f) + S_f, \tag{1}$$

$$0 = k_{s,eff} \frac{\partial^2 T_s}{\partial y^{*2}} - h_{sf} a_{sf} (T_s - T_f) + S_s. \tag{2}$$

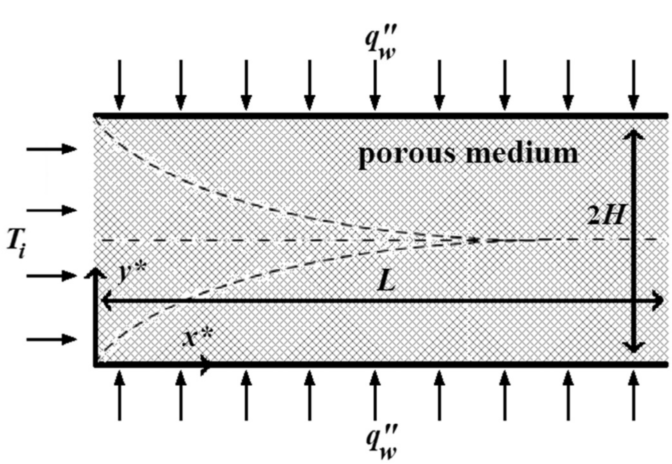

Fig. 1. Schematic diagram of the problem.

In writing Eqs. (1) and (2), it is assumed that the Peclet number is high, i.e. $Pe >> 1$ and hence the axial heat conduction term is negligible [16,20,21].

3. Nusselt number finding

3.1. Perturbation analysis (no model)

To find a relation representing the temperature difference between the solid and fluid phases in the porous material Eq. (2) is divided by $h_{sf} \times a_{sf}$:

$$(T_s - T_f) = \frac{k_{s,eff}}{h_{sf} a_{sf}} \frac{\partial^2 T_s}{\partial y^{*2}} + \frac{S_s}{h_{sf} a_{sf}} \sim O\left(\frac{1}{h_{sf} a_{sf}}\right). \tag{3}$$

It means that the temperature difference between the solid and fluid phases has the order of magnitude equal to $(h_{sf} \times a_{sf})^{-1}$. The scale analysis performed in the Appendix shows the order of magnitude of $h_{sf} \times a_{sf}$. Because the product of $h_{sf} \times a_{sf}$ is larger than unity [22–25], Eq. (3) could be rewritten as follows [26]:

$$T_s = T_f + O\left(\varepsilon = \frac{1}{h_{sf} a_{sf}}\right). \tag{4}$$

Here, ε has the order of magnitude smaller than unity. To proceed in a more general form, the following dimensionless variables and parameters are introduced:

$$\theta = \frac{(T - T_i) k_{s,eff}}{q''_w H}, \tag{5}$$

$$x = \frac{x^* k_{f,eff}}{\rho c_p H^2 u} = \frac{x^*/H}{\mathrm{Pe}}, \quad y = \frac{y^*}{H}, \quad k = \frac{k_{f,eff}}{k_{s,eff}}, \quad \omega = \frac{SH}{q''_w}, \quad Bi = \frac{h_{sf} \alpha_{sf} H^2}{k_{s,eff}}. \tag{6}$$

T_i is the inlet temperature of the fluid phase. Combining Eqs. (3)–(6) gives:

$$(\theta_s - \theta_f) = \Delta NE = \frac{1}{Bi} \frac{\partial^2 \theta_f}{\partial y^2} + \frac{1}{Bi} \omega_s + O\left(Bi^{-2}\right). \tag{7}$$

where $\Delta NE = (\theta_s - \theta_f) = (T_s - T_f) k_{s,eff} / (q''_w H)$, called the LTNE intensity. Subsequently, finding the temperature difference between the fluid and solid phases (ΔNE) is reduced to find $\partial^2 \theta_f / \partial y^2$. Adding Eq. (1) to Eq. (2) and using Eqs. (5)–(7) yield:

$$k \frac{\partial \theta_f}{\partial x} = (k+1) \frac{\partial^2 \theta_f}{\partial y^2} + \omega_f + \omega_s + O\left(Bi^{-1}\right). \tag{8}$$

For the sake of brevity, the subscript 'f' is dropped from 'θ_f' in the remainder of the text. Now, the solution is similar to solve a fluid saturated porous medium under the LTE condition (Eq. 8). The boundary conditions of a porous-filled channel under a constant heat flux imposed at the walls with negligible thickness are:

$$k_m \frac{\partial T}{\partial y^*} = -q''_w \quad \text{at } y^* = 0, \tag{9a}$$

$$\frac{\partial T}{\partial y^*} = 0 \quad \text{at } y^* = H, \tag{9b}$$

$$T = T_i \quad \text{at } x^* = 0. \tag{9c}$$

Eq. (9a) is valid since the wall has negligible thickness and with no axial conduction. Hence, all the heat flux imposed to the channel walls should be transferred to the fluid phase.

It should be pointed out that at the micro-channel wall (i.e. $y^* = 0$) the fluid temperature at the wall, $T_{f,w}$, can be different