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Extended analytical approach for electrical anisotropy of geomaterials

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ABSTRACT

We model the anisotropy of the electrical conductivity of geomaterials based on the micro–macro homogenization theory. These materials are considered as random mixtures of solid grains and pores filled by fluids, both are supposed to have ellipsoidal shapes with their long axes oriented in horizontal direction. The electrical behavior of such material is transversely isotropic. The classical Eshelby's concept of a mixture of an ellipsoidal inclusion in an infinite homogeneous matrix, that was developed to study elastic properties of heterogeneous materials, is extended to analyze the conductivity of rocks. A combination of the self-consistent and the differential effective medium techniques allows developing a theoretical formula for the simulation of conductivity of anisotropic heterogeneous materials. For particular isotropic cases, this formula is similar to the classical well-known solutions that are largely used in practice such as Archie's law, Bruggman's theory and Bussian's equation. When applying to geomaterials, the developed theory provides the conductivities in both horizontal and vertical directions. The anisotropy, defined as the ratio between these two conductivities, is a function of the porosity, the shapes and the conductivities of each phase of rocks. This paper, focusing on a purely theoretical approach, shows how the micromechanical parameters affect the macroscopic anisotropy of electrical conductivity and resistivity of anisotropic materials.

1. Introduction

Electrical conductivity is an important geophysical property of rocks and largely used in geo-sciences and petroleum-sciences such as geopressure estimation (Eaton, 1975), hydrocarbon characterization (Ellis and Singer, 2007) and mineralogy analysis (Hill and Milburn, 2003). These properties are generally anisotropic due to the natural anisotropy of the microstructure (Tabbagh and Cosenza, 2007). For example in the case of compacted claystone, clay particles and pores are flat and align with long axes perpendicular to sedimentary layering (Fig. 1). The knowledge of the electrical anisotropy is then very important for the interpretation of resistivity measurement.

Many theoretical and empirical methods were developed to study the electrical conductivity and resistivity of rocks. The most famous and classical approach is the empirical Archie's power law that was developed for isotropic clean sandstone (Archie, 1942). This law is then extended by Waxman and Smits (2003) for isotropic shaly sandstone accounting the surface conductivity of the grains. These empirical approaches are theoretically demonstrated afterward by Sen et al. (1981) and Bussian (1983) based on the classical solution of Bruggeman (1935). Bussian's result is then further developed by Revil et al. (1998) considering the behavior of different kinds of ion in the pore space. Although these theories are very developed, their applications are limited to isotropic materials.

For many decades, the micro–macro homogenization approaches based on Eshelby's solution has been used as a powerful tool to study anisotropic heterogeneous materials (Eshelby, 1957; Hornby et al., 1994; Giraud et al., 2007). The macroscopic anisotropy is affected by the anisotropy of each phase in the mixture, the shape and the orientation of the particles as well as the anisotropy of the stress acting on the considered materials. Nguyen (2014) successfully employed this technique for the simulation of electrical resistivity and conductivity of saturated and unsaturated sandstone.

This paper focuses on a theoretical model of the anisotropy of electrical conductivity and resistivity of geomaterials. Solid particles and pores are supposed to have ellipsoidal shape and lay in the horizontal direction. The vertical and horizontal conductivity are determined as function of the conductivity of water filled in the pore space and that of the solid particles as well as their shape and orientation. The anisotropy parameter is defined as a ratio of the vertical and horizontal conductivity.

We start by introducing Eshelby's solution for overall electrical conductivity of a mixture of an ellipsoidal inclusion in an infinite homogeneous matrix. This solution is then developed by combining the self-consistent (SC) and the differential effective medium (DEM) techniques to obtain the macroscopic electrical conductivity of a mixture of many components. The obtained result is, for the particular isotropic case, confirmed to be identical to the classical well-known solutions of

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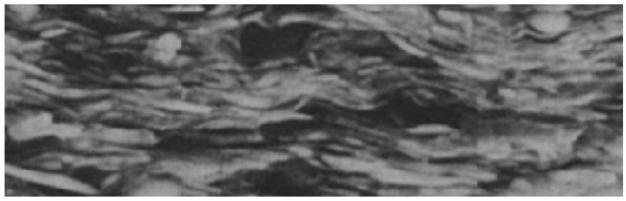

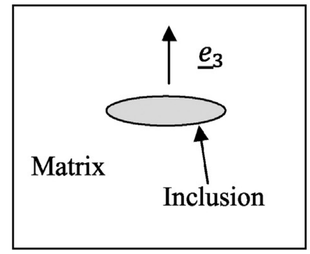

Fig. 1. Left-side: microstructure of compacted claystone (Hornby et al., 1994); Right-side: matrix-inclusion problem.

Bruggeman, Archie, Sen and Bussian. It is then applied for modeling the anisotropy of rocks. This application is purely theoretical, i.e. no real calibration is done due to the lack of experimental data of the anisotropy of electrical resistivity and conductivity of geomaterials. However it clearly demonstrates how the micromechanical parameters affect the macroscopic anisotropy of electrical conductivity and resistivity of anisotropic materials.

Notations

- $\boldsymbol{C}$ is the second order conductivity tensor
- $\boldsymbol{A}$ is the second order localization tensor
- $\boldsymbol{P}$ is the second order Hill's tensor
- $\mathbf{1}$ is the second order unit tensor
- f is the volume fraction
- C is the conductivity
- R is the resistivity
- c is the dimensionless conductivity, ratio between C and the conductivity of water
- r is the dimensionless resistivity, inverse of c
- ν and Q are the anisotropic parameters of the inclusion
- ϕ is the porosity
- V is volume of a phase in a mixture
- γ is the electrical anisotropy, ratio between vertical and horizontal resistivity

The exponents and index

- m is for the matrix phase
- i is for the inclusions
- s is for the solid phase
- w is for water

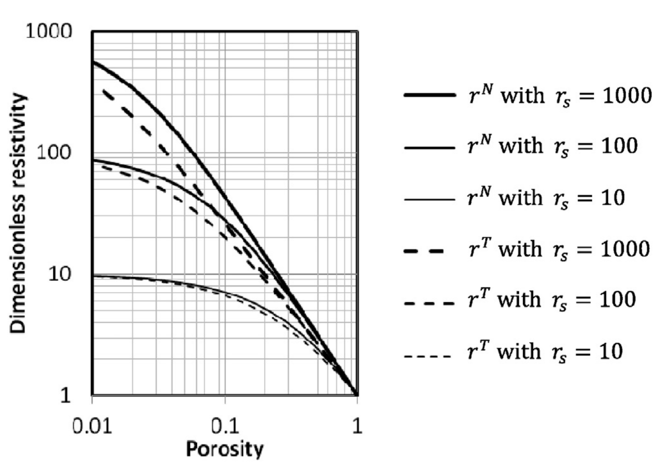

Fig. 2. Normal and transversal dimensionless resistivities versus porosity for the case of nearly spherical particles: $Q = 0.3$

- T is for transversal component of the transversely isotropic tensors
- N is for normal component of the transversely isotropic tensors
- SC is for self-consistent scheme

2. Theoretical framework

Considering a system of a single ellipsoidal inclusion in an infinite homogenous matrix (Fig. 1, left-side), it is demonstrated that the electric field localization tensor is a function of the conductivity of the matrix as well as the shape and the conductivity of the inclusion as shown by the following equation (Giraud et al., 2007; Eshelby, 1957):

$$\boldsymbol{A} = (\mathbf{1} + \boldsymbol{P}(\boldsymbol{C_i} - \boldsymbol{C_m}))^{-1} \tag{1}$$

where $\mathbf{1}$ is the second order unit tensor and $\boldsymbol{P}$ is the Hill's tensor which depends on the conductivity of the matrix C_m, the conductivity C_i and the shape of the inclusion (Hill, 1965). For the case of spheroidal transversely isotropic inclusion in transversely isotropic matrix and the anisotropy evolution direction $\underline{e}_3$ is the same for the inclusion and the matrix (Fig. 2), the Hill tensor is calculated by (Giraud et al., 2007; Nguyen, 2014):

$$\begin{aligned} &\boldsymbol{P} = P^T\left(\mathbf{1} - \underline{e}_3 \otimes \underline{e}_3\right) + P^N \underline{e}_3 \otimes \underline{e}_3 \\ &P^N = \frac{1-2Q}{C_m^N};\ P^T = \frac{Q}{C_m^T} \end{aligned} \tag{2}$$

where C_m^N and C_m^T are the conductivity of the matrix in the normal ($\underline{e}_3$) and transversal directions respectively. The parameter Q is a function of the shape and the anisotropy of the inclusion which is characterized by the parameter $\nu = X * \sqrt{C_i^N / C_i^T}$, with X the aspect ratio of the inclusion (the ratio between the width and the diameter of the spheroid). The parameters C_i^N and C_i^T are the conductivity of the inclusion.

For the case when $\nu < 1$ the parameter Q is calculated by:

$$Q = \frac{1}{2} - \frac{\sqrt{1-\nu^2} - \nu \arctan\left(\frac{\sqrt{1-\nu^2}}{\nu}\right)}{2\left(1-\nu^2\right)^{3/2}} \tag{3}$$

The parameter Q tends to 1/3 and 0 when ν tends to 1 (spherical isotropic inclusion) and 0 (disk-like inclusion) respectively.

Note that for a mixture of n components, the volume average of the localization tensor (calculated by Eq. (1)) satisfies the following condition:

$$\textstyle\sum_{k=1}^{n} f_k \boldsymbol{A}_k = \mathbf{1} \tag{4}$$

where f_k and $\boldsymbol{A}_k$ are the volume fraction and the localization tensor of a phase k.