



# Resolution equivalence of dispersion-imaging methods for noise-free high-frequency surface-wave data



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## ABSTRACT

A key step in high-frequency surface-wave methods is to acquire the dispersion properties of surface waves. To pick dispersion curves, surface-wave data is required to be transformed from the time–space ( $t$ – $x$ ) domain to the frequency–phase velocity ( $f$ – $v$ ) domain. In constructing accurate multi-mode dispersion curves, it is necessary to generate a reliable and high-resolution image of dispersion energy in the  $f$ – $v$  domain. At present, there are five methods available for imaging dispersion energy: the  $\tau$ – $p$  transformation, the  $f$ – $k$  transformation, the phase shift, the frequency decomposition and slant stacking, and the high-resolution linear Radon transformation. Among them, resolution of the high-resolution linear Radon transformation is the highest. We proposed a processing step to enhance resolution of the other four methods. Resolution of the four enhanced methods and high-resolution linear Radon transformation is compared by imaging dispersion energy of the same synthetic data. As a result, the four enhanced methods generated dispersion images with the same resolution, which revealed that the resolution of the five dispersion-imaging methods is essentially equivalent for noise-free data.

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## 1. Introduction

Surface waves are dispersive for layered earth models. Particular modes of surface waves will possess unique phase velocities at different frequencies. A key step in high-frequency surface-wave methods is to acquire the dispersion properties of surface waves. Generating a reliable and high-resolution image of dispersion energy in frequency–phase velocity ( $f$ – $v$ ) domain is of essential necessity (Xia, 2014). From a high-resolution dispersion image, accurate dispersion curves can be picked by following peaks of dispersion energy at different frequencies, leading to an effective inversion.

In high-frequency surface-wave methods, there are five methods available for imaging dispersion energy at present: the  $\tau$ – $p$  transformation (McMechan and Yedlin, 1981), the  $f$ – $k$  transformation (e.g., Yilmaz, 1987), the phase shift (Park et al., 1998), the frequency decomposition and slant stacking (Xia et al., 2007), and the high-resolution linear Radon transformation (Luo et al., 2008). Among them, high-resolution linear Radon transformation usually generates higher-resolution dispersion images than the other four methods do (Luo et al., 2008). Since the method has been utilized in many applications (Herrmann et al., 2000; Foster and Mosher, 1992; Thorson and Claerbout, 1985), it has received more and more attentions. Based on our current works,

we find that there are ways to enhance resolution of the four other methods, so that the five methods can possess the same resolution power in the  $f$ – $v$  domain. One simple practice is the numeric processing that based on power operation.

In this paper, we first introduce principles of the five dispersion-imaging methods and are solution-enhancing processing. Then dispersion energy of the same synthetic surface-wave shot gather is imaged using all the five methods. Besides, the processing step is executed to enhance resolution of dispersion images that are generated by  $\tau$ – $p$  transformation,  $f$ – $k$  transformation, phase shift, and frequency decomposition and slant stacking. Finally, we conduct a comparison to summarize resolution of the five dispersion-imaging methods.

## 2. Methods

To acquire dispersion energy in the  $f$ – $v$  domain, a shot gather in the time–space ( $t$ – $x$ ) domain is necessary to go through two steps: one is to transform from the time domain to the frequency domain; the other one is to transform from the space domain to the phase velocity domain by velocity scanning and slant stacking. These two steps are independent and they can switch their transformation sequences with each other during calculation.

Let  $s(x, t)$  be surface-wave wavefield, where  $x$  is offset (distance between a source and a receiver) and  $t$  is travel time, all receivers are evenly arranged along a survey line with a spacing  $\Delta x$ .

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### 2.1. $\tau$ - $p$ transformation (McMechan and Yedlin, 1981)

A slant stack operator is first applied to linearly transform wavefield  $s(x, t)$  into  $\tau$ - $p$  domain data:

$$U(p, \tau) = \sum_{i=1}^N s(x_i, t = \tau + px_i), \quad (1)$$

where  $p$  is the slowness and  $\tau$  is the intercept on the time axis. If the scanning slowness  $p$  is close to the real slowness of surface wave,  $U(p, \tau)$  will reach a maximum. A Fourier transformation is then applied to the data in the  $\tau$  domain to transform them into the frequency domain. As  $p$  is simply the reciprocal of phase velocity ( $p = 1/v$ ), the  $f$ - $v$  domain result can be easily converted and achieved.

### 2.2. $F$ - $k$ transformation (e.g., Yilmaz, 1987)

Wavefield  $s(x, t)$  is first transformed to the  $f$ - $k$  domain data  $S(k, f)$  by a two-dimensional Fourier transformation:

$$S(k, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s(x, t) e^{-2\pi i(ft+kx)} dt dx. \quad (2)$$

This procedure could be replaced with a one-dimensional Fourier transformation in the time domain and a one-dimensional inverse Fourier transformation in the space domain. Then the  $f$ - $v$  domain result can be transformed according to the relation:  $v = f/k$ .

### 2.3. Phase shift (Park et al., 1998)

Phase information  $P(x, \omega)$  of a wavefield can be calculated by a Fourier transformation in the time domain. It is expressed as

$$P(x, \omega) = e^{-i\frac{\omega}{v}x + \varphi_0}, \quad (3)$$

where  $\omega$  is the circular frequency,  $v$  is the phase velocity and  $\varphi_0$  is the initial phase. The phase is linearly correlated to offset. Integrate the phase using a scanning  $\varphi$ :

$$U(\varphi, \omega) = \int e^{i\varphi x} P(x, \omega) dx = \int e^{i(\varphi - \frac{\omega}{v})x + \varphi_0} dx = \sum_{i=1}^N e^{i(\varphi - \frac{\omega}{v})x_i + \varphi_0}. \quad (4)$$

When the scanning  $\varphi$  gets close to  $\omega/v$ , phase angle of the complex number  $e^{i(\varphi - \frac{\omega}{v})x + \varphi_0}$  is the same in the complex plane, making the integral reaching a maximum value. At last, the  $f$ - $v$  domain result can be achieved according to the following relations:

$$f = 2\pi\omega, \text{ and} \quad (5)$$

$$v = \frac{\omega}{\varphi} = \frac{2\pi f}{\varphi} = \frac{2\pi}{\varphi} f. \quad (6)$$

Phase shift method utilizes the normalized spectrum and can perform well in generating dispersion image (Moro et al., 2003). Nevertheless, due to the uniqueness of calculated phase at each frequency, dispersion energy of higher modes may not be completely imaged.

### 2.4. Frequency decomposition and slant stacking (Xia et al., 2007)

Shot gather  $s(x, t)$  is first stretched into pseudo-vibroseis data  $d(x, t)$  using a frequency decomposition technique (Coruh, 1985):

$$d(x, t) = y(t) \oplus s(x, t), \quad (7)$$

where  $\oplus$  stands for the convolution operator and  $y(t)$  is a sweep function. Generally,  $y(t)$  is usually chosen as a linear function  $y(t) = a + bt$ .

So  $d(x, t)$  is also a frequency-swept data  $d(x, f)$  and each  $t$  corresponds with a frequency. A slant stacking (Yilmaz, 1987, p. 430) is then performed to obtain  $f$ - $p$  domain data  $U(p, f)$ . The  $f$ - $v$  domain result can be eventually achieved by inverting  $p$ .

The calculated result of this method is better than that of  $\tau$ - $p$  transformation because frequencies are decomposed before slant stacking. Furthermore, compared with phase shift, this algorithm could extract stronger energy for higher-mode surface waves.

### 2.5. High-resolution linear Radon transformation (Luo et al., 2008)

Standard linear Radon transformation can be expressed as the following equation

$$d(x, f) = \sum_{i=1}^n m(p_i, f) e^{i2\pi f p_i x}, \quad (8)$$

where  $d(x, f)$  is a shot gather in the frequency domain and  $m(p, f)$  is the Radon matrix. So Eq. (8) can be written as a matrix form:

$$\mathbf{d} = \mathbf{Lm}, \quad (9)$$

where  $\mathbf{L} = e^{i2\pi f p x}$  is the linear Radon transformation (LRT) operator. Eq. (9) can be rearranged as

$$\mathbf{m}_{adj} = \mathbf{L}^T \mathbf{d}, \quad (10)$$

where  $\mathbf{m}_{adj}$  represents a low-resolution solution using the transpose of  $\mathbf{L}$ .

An appropriate weighting and damping factor will lead to a sparse inversion solution and improve the resolution. Thus, the objective function becomes

$$J = \|\mathbf{W}_d (\mathbf{d} - \mathbf{LW}_m^{-1} \mathbf{W}_m \mathbf{m})\| + \lambda \|\mathbf{W}_m \mathbf{m}\|^2, \quad (11)$$

and the problem turns into solving the following equation (Ji, 2006)

$$(\mathbf{W}_m^{-T} \mathbf{L}^T \mathbf{W}_d^T \mathbf{W}_d \mathbf{LW}_m^{-1} + \lambda \mathbf{I}) \mathbf{W}_m \mathbf{m} = \mathbf{W}_m^{-T} \mathbf{L}^T \mathbf{W}_d^T \mathbf{W}_d \mathbf{d}, \quad (12)$$

where  $\mathbf{I}$  is the identity matrix,  $\mathbf{W}_d$  is a data weighting matrix and  $\mathbf{W}_m$  is a model weighting matrix,  $\lambda$  is a damping factor and controls the compromise between the error misfit and model length. Eq. (12) can be solved by the weighted preconditioned conjugate gradient (CG) algorithm (e.g., Sacchi and Ulrych, 1995).

The resolution of high-resolution LRT depends on the iteration in the CG algorithm. At the beginning of iteration, this method equals with a standard LRT (only iterate once). With the number of iteration growing, the resolution will be greatly improved (normally iterate 3 times).

### 2.6. A resolution-enhancing processing

We define the resolution of a dispersion image as the width between the two half-values of dispersion energy at a given frequency in the  $f$ - $v$  domain (Xia et al., 2006).  $\tau$ - $p$  transformation,  $f$ - $k$  transformation, phase shift, frequency decomposition and slant stacking are actually kinds of a standard discretized LRT and they generate dispersion images with a low resolution (Eq. (10)). To compare them with high-resolution LRT, we deem them regular dispersion-imaging methods in this paper for convenience. They suffer from incomplete information (Trad et al., 2003) and could hardly generate high-resolution dispersion images.

The mathematical power operation is able to amplify the difference between large and small values, outstanding the maxima. Computing a

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