Journal of Applied Geophysics 118 (2015) 106–114

Contents lists available at ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

Improved curvature gravity gradient tensor with principal component analysis and its application in edge detection of gravity data

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ARTICLE INFO

Article history:
Received 22 December 2014
Received in revised form 17 April 2015
Accepted 18 April 2015
Available online 25 April 2015

Keywords:
Gravity data
Edge detection
Curvature gravity gradient tensor
Principal component analysis
Noisy data

ABSTRACT

Edge detection is one of the common processes in the interpretation of gravity data, and it gives a view of the earth which highlights boundaries of geological bodies. Many mathematical techniques have been proposed for edge detection including the curvature gravity gradient tensor method (CGGT). Eigenvalues of the curvature gravity gradient tensor can well delineate the edges of some geological bodies. However, it cannot be applied to complex gravity data with both positive and negative anomalies. Moreover, the CGGT method is also very sensitive to noise. In view of limitations of the method, we proposed an improved CGGT method by incorporating the principal component analysis (PCA) into the CGGT formula. The improved method can be utilized to outline the edges of causative bodies in more general cases and is insensitive to noise. The method was tested on synthetic gravity data and actual gravity data recorded from a metallic mineral deposit area in the middle-lower reaches of the Yangtze River in China. All of the results have shown effectiveness of the proposed method.

1. Introduction

Gravity exploration has been playing an important role in mineral resource detection. Rational interpretation of gravity data gives a view of the earth that highlights geological bodies. Edge detection is one of the common processes in the interpretation process. It brings out edges of geologic targets and visualized features of geologic structures (Nabighain et al., 2005a, 2005b; Wang et al., 2014).

Detection of the edges of gravity data is usually based on derivative transformations of the anomaly data and its properties. Over the decades, many edge detection techniques have been developed, such as zero contours of the vertical derivatives, extreme points of total horizontal derivatives and analytical signal amplitudes (Evjen, 1936; Nabighian, 1972; Cordell and Grauch, 1985; Blakely and Simpson, 1986; Mallat and Zhong, 1992; Trompat et al., 2003; Wijns et al., 2005; Cooper and Cowan, 2008).

Using of curvature attributes is a popular technique in seismic data interpretation, which measures the degree of bending of seismic reflections along a surface or in a volume, enabling identification of subtle faults, fractures, and other geological features (Roberts, 2001; Klein et al., 2008; Guo et al., 2014). Then, the curvature method is applied to the interpretation of potential field data. Hansen and deRidder (2006) have proposed an approach to depict the linear feature and depth estimation for aeromagnetic data based on the curvature of the total horizontal gradient. Phillips et al. (2007) have established a new method to interpret the potential field data using curvature attributes. Cooper (2009) has used profile curvature to detect the edges of causative sources. Oruç et al. (2013) have used the curvature gravity gradient tensor (CGGT) to interpret the geological structure of the Erzurum Basin.

With CGGT method, the edges of geological bodies are outlined by eigenvalues of the gravity gradient tensor. However, the curvature-based approach has not yet become a mainstream technique for detection of the edges of gravity anomalies for some flaws. Based on former research, we found out that it cannot be applied to complex gravity data with both positive and negative anomalies. More narrowly, the large eigenvalues of CGGT can only outline edges of geological bodies with positive density contrast, while the small eigenvalues can only delineate the edges of geological bodies with negative density contrast. Unfortunately, actual field data is general to be composed of positive anomalies as well as negative anomalies. Zhou et al. (2013) have incorporated the raw field data into the CGGT formula to make it suitable for complex gravity data, and called it the improved curvature gravity gradient tensor (ICGGT). Nevertheless, the ICGGT method is not very effective when the data is contaminated with a lot of noise. Noise in the raw data will blur the edges identified by ICGGT, and some false edges will be depicted as well.

In the work discussed in this paper, the CGGT method for detection of the edges of gravity anomalies was improved by the principal components analysis technique (PCA). PCA is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components. With PCA, we can elect principal components while neglecting small blurs in the raw data. The new

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http://dx.doi.org/10.1016/j.jappgeo.2015.04.013
0926-9851/

improved method is proposed as a mean of edge detection in more general cases. Meanwhile, it is not very sensitive to noise in the data. The new approach was tested by using both synthetic gravity anomaly data and real gravity data from a metallic deposit area in the middle-lower reaches of the Yangtze River in China.

2. Analysis of the conventional CGGT method

The derivation of Hessian matrix of the horizontal vector gradients of gravity data is made by Hansen and deRidder (2006) for magnetic applications. The Hessian matrix is also called curvature which describes how bent a curve or surface is at a particular point on a geometric curve or surface (Oruç et al., 2013). The curvature gradient matrix of the gravity field is defined as Eq. (1),

$$\mathbf{G} = \begin{pmatrix} \frac{\partial g_x}{\partial x} & \frac{\partial g_x}{\partial y} \\ \frac{\partial g_y}{\partial x} & \frac{\partial g_y}{\partial y} \end{pmatrix} \tag{1}$$

where g_x is the gravity data, g_x and g_y are the horizontal gravity vectors in x and y directions, $\frac{\partial g_x}{\partial x}$, $\frac{\partial g_x}{\partial y}$, $\frac{\partial g_y}{\partial x}$ and $\frac{\partial g_y}{\partial y}$ denote the first order derivatives of g_x and g_y in x and y directions respectively. Matrix **G** is symmetric because of the relationship $\frac{\partial g_x}{\partial y} = \frac{\partial g_y}{\partial x}$. And it can be represented with a diagonal matrix whose components are the eigenvalues of Hessian matrix, and we write it as Eq. (2) (Boring, 1998),

$$\mathbf{\Lambda} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \tag{2}$$

where λ_1 and λ_2 are the eigenvalues. Then, Eq. (3)

$$(\mathbf{G} - \mathbf{\Lambda I})x = 0 \tag{3}$$

is utilized to compute λ_1 and λ_2, where x denotes the eigenvectors of matrix that corresponds to the particular eigenvalues. The eigenvalues of matrix **G** can be written as Eq. (4),

$$\begin{aligned} \lambda_1 &= 0.5 \cdot \left(\frac{\partial g_x}{\partial x} + \frac{\partial g_y}{\partial y} + \sqrt{\left(\frac{\partial g_x}{\partial x} - \frac{\partial g_y}{\partial y} \right)^2 + 4 \cdot \frac{\partial g_x}{\partial y} \cdot \frac{\partial g_y}{\partial x}} \right) \\ \lambda_2 &= 0.5 \cdot \left(\frac{\partial g_x}{\partial x} + \frac{\partial g_y}{\partial y} - \sqrt{\left(\frac{\partial g_x}{\partial x} - \frac{\partial g_y}{\partial y} \right)^2 + 4 \cdot \frac{\partial g_x}{\partial y} \cdot \frac{\partial g_y}{\partial x}} \right) \end{aligned} \tag{4}$$

With Eq. (4), we can calculate the eigenvalues λ_1 and λ_2. Then, we plot the zero-contour of the eigenvalues, and the zero-contour can delineate the edges of causative bodies. In this sense, the eigenvalues are similar to vertical derivative of the gravity anomaly, and they can be used to detect edges of gravity data.

However, numerical tests have proven that the CGGT method cannot be applied to complex gravity data with both positive and negative anomalies. More narrowly, the large eigenvalues of CGGT can only outline edges of geological bodies with positive density contrast, while the small eigenvalues can only delineate edges of geological bodies with negative density contrast. Moreover, the conventional CGGT is also very sensitive to noise. Zhou et al. (2013) have incorporated the raw field data into the CGGT formula to make it suitable for complex gravity data, and it is written as

$$\lambda = \mathbf{Data} \cdot \left(\frac{\partial g_x}{\partial x} + \frac{\partial g_y}{\partial y} + \sqrt{\left(\frac{\partial g_x}{\partial x} - \frac{\partial g_y}{\partial y} \right)^2 + 4 \cdot \frac{\partial g_x}{\partial y} \cdot \frac{\partial g_y}{\partial x}} \right) \tag{5}$$

where matrix **Data** represents the raw field data. The improved CGGT method can delineate edges of geological bodies with both positive density contrast and negative density contrast simultaneously. Based on model experiments, we find that both the CGGT and the ICGGT are sensitive to the effect of random noise. The detected edges can easily be blurred by the noise contained.

3. The improved CGGT method with PCA

In this section, we utilized principal components analysis (PCA) technique to improve the conventional CGGT. PCA was invented in 1901 by Karl Pearson (Pearson, 1901), and it is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components. The number of principal components is less than or equal to the number of original variables. This transformation is defined in such a way that the first principal component accounts for as much of the variability in the data as possible, and each succeeding component in turn has the highest variance possible under the constraint that it is orthogonal to the preceding components (Abdi and Williams, 2010). We applied the PCA into the formula of conventional CGGT method, and we call it the improved curvature gravity gradient tensor with principal component analysis (PCICGGT).

In the presented paper, we used singular value decomposition (SVD) to perform PCA. Assume the raw data is represented with a two dimensional $m \times n$ matrix **X** (it can be thought as the raw data laid out in a 2D grid), then it can be decomposed by SVD, i.e. Eq. (6):

$$\mathbf{X} = \mathbf{U \Gamma V}^T \tag{6}$$

where **U** is an $m \times m$ matrix, $\mathbf{\Gamma}$ is an $m \times n$ rectangular diagonal matrix with non-negative real numbers on the diagonal, and **V** is an $n \times n$ matrix (Gilbert, 1980). The diagonal entries of $\mathbf{\Gamma}$ are the singular values of matrix **X**. In PCA we find the directions in the data with the most variation, i.e. the eigenvectors corresponding to the largest eigenvalues of the covariance matrix, and project the data onto these directions. The eigenvectors corresponding to the larger singular values in matrix $\mathbf{\Gamma}$ are the directions in the data with the most variation. The principal components in raw data can be constituted with the relatively lager singular values and the eigenvectors in matrices **U** and **V**.

At this point, the proposed PCICGGT is defined as a polynomial named τ (Eq. (7)),

$$\tau = \max(\Gamma_1, \Gamma_2, \ldots, \Gamma_k) \cdot \mathbf{U}_k \mathbf{\Gamma}_k \mathbf{V}_k^T \cdot \left(\mathbf{A} + \mathbf{B} + \sqrt{(\mathbf{A} - \mathbf{B})^2 + 4\mathbf{C}} \right) \tag{7}$$

where $\mathbf{U}_k$, $\mathbf{V}_k$ and $\mathbf{\Gamma}_k$ are the first k columns and rows of matrices **U**, **V** and $\mathbf{\Gamma}$ in Eq. (6) respectively, which means that matrices $\mathbf{U}_k$ and $\mathbf{V}_k$ are obtained by setting all but the first k columns and rows equal to zero and matrix $\mathbf{\Gamma}_k$ is acquired by setting all but the first k largest singular values equal to zero. $\mathbf{\Gamma}_k$ is the kth singular value of the initial singular values of data matrix (arranged in decreasing order). The square root in Eq. (7) is conducted with element-wise square root. Matrices **A**, **B** and **C** are given in Eqs. (8)–(10).

$$\mathbf{A} = \frac{\partial^2 \left(\mathbf{U}_k \mathbf{\Gamma}_k \mathbf{V}_k^T \right)}{\partial x^2} \tag{8}$$

$$\mathbf{B} = \frac{\partial^2 \left(\mathbf{U}_k \mathbf{\Gamma}_k \mathbf{V}_k^T \right)}{\partial y^2} \tag{9}$$

$$\mathbf{C} = \frac{\partial^2 \left(\mathbf{U}_k \mathbf{\Gamma}_k \mathbf{V}_k^T \right)}{\partial x \partial y} \tag{10}$$