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Research Paper

Unsteady natural convection and heat transfer in a differentially heated cavity with a fin for high Rayleigh numbers

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HIGHLIGHTS

- Natural convection induced by a fin for high Rayleigh numbers is investigated.
- Transient flows are observed for different Rayleigh numbers and fin positions.
- Simple scaling argument is applied for transient flows at fin positions.
- Dependence of heat transfer on the Rayleigh number and fin position is quantified.

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ABSTRACT

Unsteady natural convection and heat transfer in a differentially heated cavity with a fin at different positions are numerically investigated for a wide range of high Rayleigh numbers from 10^8 to 10^{11} and a Prandtl number of Pr = 6.63. The results show that the development of natural convection from the start-up is dependent on the Rayleigh number and the fin position. The transient features of natural convection flows in the cavity are described. A simple scaling analysis is applied for the transient flow around the fin and the dependence of the unsteady flow on the Rayleigh number and the fin position is quantified. Heat transfer through the cavity is examined.

1. Introduction

Natural convection in a differentially heated cavity is present in a great number of industrial systems. The study of natural convection in the cavity is therefore of fundamental interest and of practical significance.

Indeed, natural convection in the differentially heated cavity has received considerable attention in the literature. One of the earliest studies was presented by Batchelor [1] based on a prototype of double pane windows. It has been demonstrated that conduction dominates heat transfer through the cavity for low Rayleigh numbers (e.g. $<10^3$) but convective heat transfer is dominant for higher Rayleigh numbers. If the Rayleigh number is sufficiently large, distinct thermal boundary layers adjacent to the cooled and heated sidewalls are formed and the fluid in the core becomes stratified at the steady state [2,3]. As the Rayleigh number increases further (e.g. larger than a critical value), the flow may become time-periodic [4–6] and even turbulent [7,8]. The majority of the early work focused on fully developed flows. However, due to the extensive presence of transient forcing, transient natural convection in the differentially heated cavity following sudden heating and cooling has been paid increasing attention. Fundamental scaling relations quantifying transient natural convection, including the development of the thermal boundary layer adjacent to the sidewall and the horizontal intrusion flow along the horizontal wall, were obtained by Patterson and Imberger [9]. Furthermore, the transient features of natural convection in the cavity such as the leading edge effect (LEE) in the thermal boundary layers and the trailing waves of the horizontal intrusion were investigated [10,11].

In addition to the above-mentioned studies of a basic model of differentially heated cavities, the study of natural convection in a differentially heated cavity with a fin on the sidewall has also been given considerable attention [12–14]. In fact, one single fin [15–17] or the array of fins [18,19] has been extensively used to control heat transfer in industry. Further, flows and heat transfer induced by micro-pin fin structures [20,21] or a fin in the electric or radiation field [22,23] have been investigated.

Heat transfer between the two differentially heated sidewalls in the cavity may be changed by the fin on the sidewall, and an understanding of the corresponding mechanisms is essential particularly for optimizing heat transfer determined by the fin

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[24–27]. Recently, the studies [28–33] show that the development of natural convection induced by a fin on the sidewall of the differentially heated cavity following sudden heating and cooling includes three stages: an early stage, a transitional stage, and a fully developed stage. In the early stage, the fin may block the vertical thermal boundary layer flow and force it to detach from the finned sidewall and thus a lower intrusion front results. The intrusion under the fin reattaches to the downstream sidewall after it bypasses the fin. A double-layer structure of the thermal boundary layer appears in the transitional stage and is ultimately formed in the fully developed stage. It has been demonstrated that the fin influences laminar natural convection flows in the cavity [12–14] and may even trigger the transition to unsteady natural convection flows in the cavity [29,30]. That is, the transition to a periodic flow around the fin in a water-filled cavity may happen if the Rayleigh number is sufficiently large. The critical Rayleigh number is also dependent on the fin length (the fin thickness is often considered to be negligibly small in comparison with its length, i.e. so-called thin fin, refer to References 12–14 for details). The studies show that the flow separation and oscillations of the thermal flow around the fin may in turn trigger traveling waves in the thermal boundary layer downstream of the fin [29–32,34]. As a consequence, heat transfer through the water-filled cavity is significantly enhanced by up to 23%.

The study by Le Quéré and Behnia [6] shows that natural convection in an air-filled cavity without a fin is unsteady if $\mathrm{Ra} \geq 1.82 \times 10^8$. However, the recent study [35] has demonstrated that the transition to an unsteady flow in the air-filled cavity with a fin may happen at a much smaller Rayleigh number ($\mathrm{Ra} \geq 3.73 \times 10^6$). The results in Reference 35 further show the distinct variations between natural convection flows in the air-filled and the water-filled cavity with a fin; that is, the effect of the Prandtl number is significant on natural convection in the cavity with a fin.

The previous studies of natural convection in the cavity with a fin have been in the context of the fin length and position (also see Reference 34) and the Prandtl number. However, inspired by heat transfer enhancement by the fin-induced oscillations, this study considers unsteady natural convection in the cavity with a fin for a wide range of high Rayleigh numbers up to 10^{11}. This is because unsteady natural convection induced by the fin may improve heat transfer through the sidewall downstream of the fin. That is, it is of practical significance to investigate how the convective flow and heat transfer in the cavity are improved for high Rayleigh numbers and the dependence on the Rayleigh number of the frequency of the oscillations and heat transfer in the cavity with a fin at different positions on the sidewall. These motivate the present numerical study.

In the remainder of this paper, the numerical procedures are described in Section 2; transient features of natural convection in the cavity with a fin at different positions from the start-up for high Rayleigh numbers are characterized and quantified in Section 3; heat transfer through the finned sidewall is discussed in Section 4; and Section 5 summarizes the major conclusions drawn from this study.

2. Numerical procedures

Under consideration is a two dimensional cavity (height H and width L), as shown in Fig. 1. The two fins are horizontally placed on the sidewalls, respectively. Their positions are symmetrical about the center of the cavity. Therefore, the flows adjacent to the two sidewalls are also approximately symmetrical about the center of the cavity and in turn heat into the cavity may balance heat out of the cavity. The top and bottom walls and the fins are adiabatic; the two sidewalls are isothermal and fixed at T_c and T_h, respectively; and all interior walls and the fins are rigid and no-slip. The working fluid is initially quiescent. At $t = 0$, the temperature of the fluid is T_0. The development of natural convection in the cavity is governed

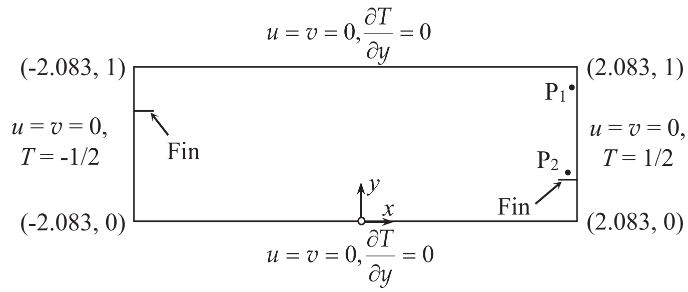

Fig. 1. Schematic of the computation domain and boundary conditions. Here, the point P_1 is at ($x = 2.075$, $y = 0.917$) and the point P_2 is downstream of the fin ($x = 2$) with a distance of 0.05 to the fin, which are recording points used in the subsequent figures.

by the following dimensionless two-dimensional Navier–Stokes and energy equation with the Boussinesq approximation (also see Reference 30):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\mathrm{Pr}}{\mathrm{Ra}^{1/2}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\mathrm{Pr}}{\mathrm{Ra}^{1/2}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \mathrm{Pr}\,T, \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\mathrm{Ra}^{1/2}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

where the origin of the coordinates is at the center of the cavity (refer to Fig. 1). The length, time and temperature difference (T-T_0) are non-dimensionalized by H, $H^2\kappa^{-1}\mathrm{Ra}^{-1/2}$ and ΔT ($=T_h - T_c$), respectively. The three dimensionless parameters which govern the flow are the Rayleigh number (Ra), the Prandtl number (Pr) and the aspect ratio (A), defined as (see e.g. Reference 1 for details)

$$\mathrm{Ra} = \frac{g\beta(T_h - T_c)H^3}{\nu\kappa}, \tag{5}$$

$$\mathrm{Pr} = \frac{\nu}{\kappa}, \tag{6}$$

$$A = \frac{H}{L}. \tag{7}$$

It is worth noting that in this study, the working fluid is water with a constant Prandtl number of $\mathrm{Pr} = 6.63$; the aspect ratio of the overall cavity is $A = 0.24$ and the fin length is $l_f = 1/6$ for comparison with the previous experiment [29]. In order to observe the effect of the Rayleigh number and the fin position, a number of numerical simulations focusing on natural convection in the cavity with the fin at different positions ($y_f = 1/6$ to $5/6$) for $\mathrm{Ra} = 10^8$ to 10^{11} were performed.

The governing equations were implicitly solved by using a finite-volume SIMPLE algorithm. The advection terms were discretized by a QUICK scheme, and the time integration was by a second-order backward difference method (also see References 30,31). The above numerical procedures were performed using FLUENT software.

Two non-uniform grid systems (479×300 and 749×450) with coarser grids in the core and finer grids concentrated in the proximity of all wall and fin boundaries (see Fig. 2a) were constructed for grid dependence tests. The tests were performed for the