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Second-order seismic wave simulation in the presence of a free-surface by pseudo-spectral method

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ABSTRACT

In this paper second-order seismic wave equations are adopted to simulate the wave propagation in the presence of a free-surface. In the simulations, horizontal and vertical spatial derivatives are solved by Fourier and Chebyshev pseudo-spectral algorithms respectively. In addition, Fourier–Chebyshev pseudo-spectral method is also used to solve the boundary conditions of the free-surface, which are Neumann boundaries. That makes results at the free-surface keep same high precision with inner domain. The new scheme is tested against Lamb's problem in a uniform elastic half space. Agreement between numerical and analytical results is every good. Compared with the former simulation scheme based first-order velocity–stress equations, the new scheme based on second-order equations is more efficient because it has fewer controlling equations.

1. Introduction

A free-surface always has great effects on seismic propagation underground, such as strong reflection of seismic wave, generation of a Rayleigh wave. So free-surface problems have been one spotlight in geophysics, and many scholars have used finite element and finite difference methods to model seismic wave propagation in the presence of a free-surface.

Different numerical methods are adopted to simulate seismic wave propagation at the presence of a free surface. Drake (1972) and Zerwer et al. (2002) implemented finite element method to simulate the seismic wave propagation along a free-surface. This method is very suitable for wave simulation in the presence of a complex free-surface but demands much computer memory and time (Wang and Zhang, 2004). Finite difference is the most frequently-used method for seismic wave modeling in the presence of a free-surface (Levander, 1988; Graves, 1996). This method is characterized by high computing speed. However, solution precision is low at the free-surface.

Pseudo-spectral method is another effective method for seismic simulation as its high precision. Usually the solution precision is guaranteed by only 2 or 3 sampling points in one wavelength (Che et al., 2010). But it has seldom been used in wave simulation with a free-surface because the corresponding boundary conditions are hard to treat. Recently a new Chebyshev pseudo-spectral method is implemented to model wave propagation in the presence of a free-surface (Kosloff et al., 1990; Tessmer and Kosloff, 1994; Carcione and Wang, 1993; Carcione, 1996). The first-order velocity–stress equations of seismic wave are taken as controlling equations in these works. Because the corresponding boundary conditions are Dirichlet conditions and easily imposed with zero normal stresses at the free surface. But 5 equations in the first-order controlling equations may increase the cost of numerical simulation. An ever-present challenge is to increase the efficiency without decreasing accuracy.

To deal with the challenge, second-order seismic equations are adopted as controlling equations for seismic wave simulation in the presence of a free-surface by a Fourier–Chebyshev pseudo-spectral method in this paper. In the equations, the horizontal and vertical spatial derivatives are solved by Fourier and Chebyshev pseudo-spectral algorithms respectively. While the free-surface conditions, which are Neumann boundary conditions corresponding to second-order wave equations, are also dealt with Fourier–Chebyshev pseudo-spectral method.

The other parts of this paper are organized as follows. Part 2 introduce second-order seismic wave equations and the corresponding Neumann boundary conditions at a free-surface. Part 3 illustrate the matrix expressions of Fourier and Chebyshev pseudo-spectral method, which are going to treat the boundary conditions of a free-surface later. In Part 4 we explain the numerical modeling scheme, especially the treatment for the boundary conditions at the free-surface. In Part 5 Lamb's problem in a uniform elastic half space is solved to test the modeling method. The wave field of a weathered-layer model is simulated by the new scheme based on second-order controlling equations and the former scheme based on first-order ones respectively. A comparison of computation cost is made on these simulations. Part 6 summarizes the characters of new modeling scheme by Fourier–Chebyshev pseudo-spectral method based on second-order equations. In addition, Fourier

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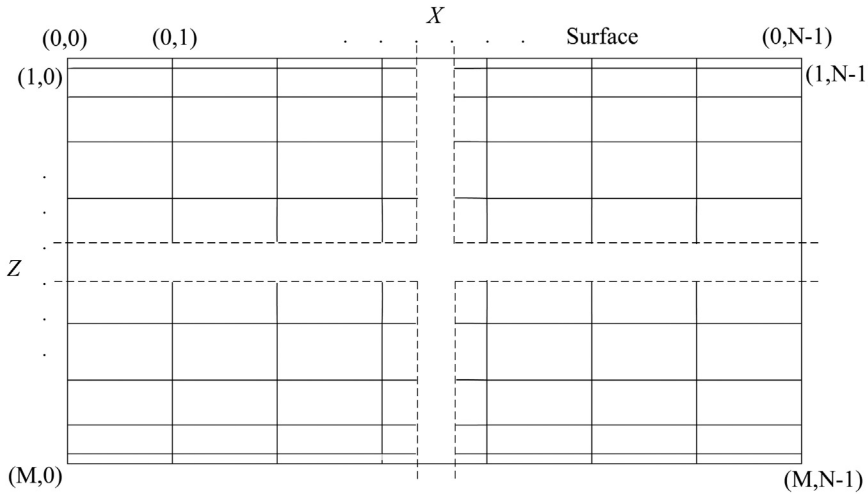

Fig. 1. The discrete grids of Fourier–Chebyshev pseudo-spectral method in 2-D space.

and Chebyshev pseudo-spectral method are chiefly introduced in Appendix A.

2. Problem formulations

Numerical simulation of wave propagation is indeed the process of solving its governing equations and the corresponding boundary conditions. The second-order seismic wave equations and the corresponding boundary conditions at a free-surface are shown in this section.

2.1. Second-order seismic wave equations

Numerical simulation of seismic wave propagation is based on the equations of conservation of momentum combined with stress–strain relation for elastic medium underground (Fung, 1965). For isotropic and piecewise homogeneous medium, substituting stress–strain relation into equations of conservation of momentum, we have the second-order displacement equations. In a 2-D medium with a horizontal axis x and a vertical axis z pointing downwards, the second-order equations, the basis of simulation in this paper, are (Yang, 2012)

$$\begin{aligned} \ddot{u}_x &= V_p^2 \frac{\partial^2 u_x}{\partial x^2} + \left(V_p^2 - V_s^2\right) \frac{\partial^2 u_z}{\partial x \partial z} + V_s^2 \frac{\partial^2 u_x}{\partial z^2} + f_x, \\ \ddot{u}_z &= V_p^2 \frac{\partial^2 u_z}{\partial z^2} + \left(V_p^2 - V_s^2\right) \frac{\partial^2 u_x}{\partial x \partial z} + V_s^2 \frac{\partial^2 u_z}{\partial x^2} + f_z \end{aligned} \tag{1}$$

where u_x and u_z are horizontal and vertical displacement components, V_p and V_s are velocities of P-wave and S-wave, f_x and f_z represent volume forces per unit mass, and a dot above a variable denotes time differentiation.

However, only when boundary conditions at the free surface are specified can the second-order governing equations determine the history of displacements of particles underground. So we propose the following equations to specify boundary conditions.

2.2. The boundary conditions at a free-surface

According to elasticity, when seismic wave propagates in the earth the traction forces at a free-surface are zero. Assuming the free-surface is flat, the corresponding boundary conditions are

$$\begin{aligned} \sigma_z &= 0, \\ \tau_{xz} &= 0 \end{aligned} \tag{2}$$

where σ_z and τ_{xz} are stresses. For u_x and u_z, the variables in the governing equations, Eq. (2), are rewritten into

$$\begin{aligned} & V_p^2 \frac{\partial u_z}{\partial z} + \left(V_p^2 - 2V_s^2\right) \frac{\partial u_x}{\partial x} = 0 \\ & \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} = 0. \end{aligned} \tag{3}$$

As elliptic partial differential equations, they are also solved by pseudo-spectral method to achieve high precision at the free-surface as the inner computation domain.

3. Matrix operators of Fourier and Chebyshev pseudo-spectral method

Because of high precision and high efficiency method, Fourier–Chebyshev pseudo-spectral method is addressed to the second-order equations and boundary conditions equations. As to the second-order equations, we approach the horizontal differentials numerically by Fourier pseudo-spectral method (more details in Appendix A) and vertical differentials by Chebyshev pseudo-spectral method (more details in Appendix A). The conventional pseudo-spectral method is suitable for hyperbolic partial differential equations with time derivatives, such as Eq. (2). Thus, as to the boundary conditions equations, which belong to elliptic partial differential equations, we present matrix operators of Fourier and Chebyshev pseudo-spectral methods. For the horizontal and vertical differentials, we use matrix operators of Fourier pseudo-spectral method and Chebyshev pseudo-spectral method respectively.

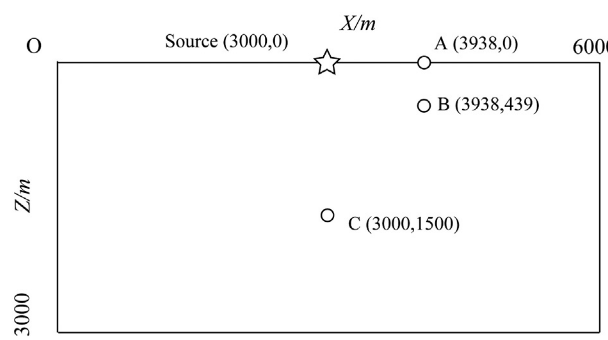

Fig. 2. Homogeneous model of Lamb's problem.