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Fast seismic horizon reconstruction based on local dip transformation

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ABSTRACT

We propose a fast and real-time interactive method to reconstruct a seismic horizon with respect to a set of picked input points. The reconstruction domain is subdivided in quadrilateral domains which are determined from input points while the entire horizon is obtained part-by-part by juxtaposing independent partial reconstructions. Each quadrilateral domain is mapped onto a rectangular domain on which a non-linear partial derivative equation relied on local dip is solved by an iterative process based on a Poisson equation. The key point is the transformation of the local dip which allows the carrying out of a direct Fourier method with a low computational cost.

1. Introduction

Seismic horizon reconstruction has become a leading method to improve seismic data interpretation and to understand geological processes. A seismic horizon is a hypersurface of a seismic image which delimits geological layers. Many recent numerical frameworks have been dedicated to the reconstruction of a unique horizon (Bienati and Spagnolini, 1998; Blinov and Petrou, 2005; Lomask and Guitton, 2006; Lomask et al., 2006; Zinck et al., 2011) as well as to the jointed reconstruction of a set of horizons (Fomel, 2010; Ligtenberg et al., 2006; Parks, 2010). Application scopes cover various domains, like flattening (Lomask and Guitton, 2006; Parks, 2010), geological model building and reservoir characterization (Hoyes and Cheret, 2011) or chrono-stratigraphic interpretation (Donias et al., 2001; Pauget et al., 2009).

Here we focus on the reconstruction of one horizon by taking into account picked points. In case of a unique point, an efficient method proposed by (Lomask et al., 2006) is based on a two-dimensional (2-D) non-linear partial derivative equation (PDE) relied on a local dip. The PDE is solved using a Gauss–Newton approach by an iterative algorithm whose crucial step is the resolution of a Poisson equation. The extension of the method to several points by Lomask and Guitton (2006) has a computational cost which is often prohibitive for large data volume. Firstly, fast algorithms to solve the Poisson equation are irrelevant. Secondly, a reestimation of the entire horizon is required when adding or displacing a point. Moreover, this global method has to be initialized with a horizon close to the solution.

In this paper, we present an alternative local method to reconstruct a horizon with respect to a set of picked input points. Based on Lomask's iterative algorithm (2006), the approach consists in a part-by-part reconstruction. The reconstruction domain is subdivided in areas on which parts of the horizon are reconstructed independently from each other while the entire horizon is obtained by juxtaposing all reconstructed parts. The approach leads to a fast and interactive reconstruction even though it is naturally suboptimal. Indeed, fast algorithms are used to solve the Poisson equation while a real-time partial (or incremental) reestimation can be carried out: only the subdomains connected to an added or displaced point need to be recalculated. The continuity of the parts of the horizon is ensured by fixing the same values on the boundaries shared by neighboring subdomains. Moreover, fixing values on all boundaries limits the largest reconstruction domain to the convex envelope of the input points.

In the case of a 2-D domain, according to Hockney (1965), a fast reconstruction can be performed on a rectangle. In the following sections, we focus on a new fast reconstruction method for domains diffeomorphic to a rectangle and here called pseudo-rectangular domains. Each pseudo-rectangular domain is mapped onto a rectangular domain through a geometrical transformation. Instead of modifying the Poisson equation as described in standard methods (Bellman and Casti, 1971; Zhong and He, 1998), the key point of our approach is the transformation of the local dip. The Poisson equation is therefore solved by a direct Fourier method which guarantees a low computational cost.

It can be noted that our approach is valid outside the seismic application scope to reconstruct explicit surfaces of finite-dimensional vector spaces such as fibrous composite images. Moreover, the local dip transformation can be extended to reconstruct implicit surfaces of finite-dimensional vector spaces (Zinck et al., 2012).

This article is organized as follows: Section 2 introduces Lomask's horizon reconstruction algorithm, Section 3 deals with the new fast

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reconstruction method on pseudo-rectangular domains while the two last sections respectively describe the part-by-part horizon reconstruction approach and exhibit results.

2. Horizon reconstruction algorithm

A seismic horizon can be considered as a curved segment in a two-dimensional space or as a surface in a three-dimensional (3-D) space and is represented by a function f defined on a domain Ω. The function f is connected (Lomask et al., 2006) to the tangent **p** of the local dip[1] by a PDE:

$$\forall \mathrm{x} \in \Omega, \quad \nabla f(\mathrm{x}) = \mathbf{p}(\mathrm{x}, f(\mathrm{x})), \tag{1}$$

where ∇ denotes the gradient operator. In a 2-D (resp. 3-D) space, x denotes a one-dimensional (1-D) variable x (resp. a two-dimensional variable (x_1, x_2)) while the local dip is a known one (resp. two)-dimensional vector giving the slope of the horizon tangent line (resp. plane) compared to the space axis $\vec{x}$ (resp. $\vec{x}_1$ and $\vec{x}_2$). The functions f and **p** are respectively considered of class C^2 and C^1.

The horizon is obtained by solving a constrained optimization problem:

$$f = \arg\min_{g \in C^2} \int_\Omega ||\nabla g(\mathrm{x}) - \mathbf{p}(\mathrm{x}, g(\mathrm{x}))||^2 dx, \tag{2}$$

assuming that either the horizon boundary or points belonging to the horizon are known. Eq. (2) is non-linear, thus an iterative algorithm is used to solve it (Lomask et al., 2006). The horizon is initialized with a function f_0 and the iterative step is made of three parts: residual computation, update term computation and updating.

- *Residual computation*:

$$\forall \ \mathrm{x} \in \Omega, \quad \mathbf{r}_k(\mathrm{x}) = \nabla f_k(\mathrm{x}) - \mathbf{p}(\mathrm{x}, f_k(\mathrm{x})). \tag{3}$$

- *Update term computation*:

$$\delta f_k = \arg\min_{g \in C^2} \int_\Omega ||\nabla g(\mathrm{x}) + \mathbf{r}_k(\mathrm{x})||^2 d\mathrm{x}. \tag{4}$$

The solution of Eq. (4) is obtained by solving a Poisson equation

$$\Delta(\delta f_k) = -\,\mathrm{div}(\mathbf{r}_k), \tag{5}$$

where Δ denotes the Laplace operator and div is the divergence operator. Eq. (5) is associated with conditions on a subdomain Ω_I of Ω:

$$\begin{array}{l} \forall \ \mathrm{x} \in \Omega_I, \quad \delta f_0(\mathrm{x}) = f(\mathrm{x}) - f_0(\mathrm{x}) \\ \quad \text{and} \quad \delta f_k(\mathrm{x}) = 0 \quad \forall \ k > 0. \end{array} \tag{6}$$

If the horizon boundary is known, the subdomain Ω_I corresponds to the boundary $\partial\Omega$ of Ω. The problem defined by Eqs. (5) and (6) is then called boundary problem. If one or several points belonging to the horizon are known, the subdomain Ω_I corresponds to the union set of all known points. The problem defined by Eqs. (5) and (6) is then called “inner” problem.

- *Updating*:

$$\forall \ \mathrm{x} \in \Omega, \quad f_{k+1}(\mathrm{x}) = f_k(\mathrm{x}) + \delta f_k(\mathrm{x}). \tag{7}$$

Usual stopping criteria consider the norm of the residual (Lomask et al., 2006). Moreover, a maximal number K of iterations is generally fixed to ensure the algorithm stopping.

The ability to compute the update term determines the computational efficiency of the reconstruction method. On a 1-D domain and a 2-D rectangular domain, fast Fourier algorithms (Hockney, 1965) can be applied to solve boundary problems. The update term is computed in one step:

$$\delta f_k = \mathrm{FT}^{-1}\left[\frac{\mathrm{FT}[-\,\mathrm{div}(\mathbf{r}_k)]}{\mathrm{FT}[\Delta]}\right], \tag{8}$$

where FT and FT^{-1} denote respectively the Fourier transform and the inverse Fourier transform. If a unique point belonging to the horizon is known with coordinates $(\mathrm{x}^P, f(\mathrm{x}^P))$, fast algorithms can also be used by replacing condition (6) on the value of δf_k at the known point by an equivalent condition on the mean value of δf_k:

$$\begin{array}{l} \forall \ k \geq 0, \quad <\delta f_k> = \int_\Omega \delta f_k(\mathrm{x}) d\mathrm{x} \\ \quad \text{fixed such as} \quad f_k\left(\mathrm{x}^P\right) = f\left(\mathrm{x}^P\right). \end{array} \tag{9}$$

The problem defined by Eqs. (5) and (9) is then called mean problem. However, the Fourier algorithms cannot be carried out to solve the inner problem for several known points on the aforementioned domains. Iterative methods like descent direction approaches and relaxation algorithms are therefore proposed in the literature (Polyanin, 2002). On 2-D non-rectangular domains, excepted on a disk (Swarztrauber and Sweet, 1973), all problems lead to complex matrix inversions. For pseudo-rectangular domains, an alternative method is to map the physical domain Ω onto a rectangular computational domain Ω′ by introducing a diffeomorphic transformation (Bellman and Casti, 1971; Zhong and He, 1998). On the domain Ω′, a differential operator with variable coefficients takes place of the Laplace operator in Eq. (5). Matrix methods to solve Eq. (5) on Ω′ (Johansen and Colella, 1998; Leveque and Li, 1994) are relatively slow although they are less complex than the approaches previously described on Ω whereas Fourier algorithms are irrelevant.

3. Fast reconstruction on pseudo-rectangular domains

3.1. Local dip transformation

In this section, we present a fast horizon reconstruction on a pseudo-rectangular domain, assuming that either the horizon boundary or a unique point belonging to the horizon is known. Instead of replacing the Laplace operator in Eq. (5), the right term $-\,\mathrm{div}(\mathbf{r}_k)$ is modified by a local dip transformation. The boundary and mean problems can then be solved by a Fourier algorithm.

We propose to apply on Eq. (1) the diffeomorphic transformation $\mathcal{F}$ which maps the pseudo-rectangular domain Ω onto a rectangular domain Ω′. The transformation is defined by:

$$\forall \ \mathrm{x} \in \Omega, \ \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathcal{F}(\mathrm{x}) = \begin{bmatrix} \mathcal{F}_1(\mathrm{x}) \\ \mathcal{F}_2(\mathrm{x}) \end{bmatrix} \in \Omega'. \tag{10}$$

The gradient field of the function f is consequently relied on a vector field by a PDE:

$$\forall \ \mathrm{y} \in \Omega', \quad \nabla f(\mathrm{y}) = \mathbf{p}'(\mathrm{y}, f(\mathrm{y})), \tag{11}$$

where y denotes the 2-D variable (y_1, y_2). The 2-D vector $\mathbf{p}'$ is the tangent of the transformed local dip, which gives the slope of the horizon tangent plane compared to the axis $\vec{y}_1$ and $\vec{y}_2$ of Ω′. It is expressed by:

$$\mathbf{p}' = \left[J_{\mathcal{F}}{}^T\right]^{-1} \mathbf{p}, \tag{12}$$

[1] The tangent **p** is previously computed over the entire seismic data by estimating the gradient field. A principal component analysis (Marfurt, 2006) is used in our method while a plane-wave destruction algorithm (Fomel, 2002) is applied in the implementation of Lomask et al. (2006).