Journal of Applied Geophysics 96 (2013) 28–32

Contents lists available at ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

Numerical aspects of EGM08-based geoid computations in Fennoscandia regarding the applied reference surface and error propagation

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A R T I C L E I N F O

Article history:
Received 8 April 2013
Accepted 24 June 2013
Available online 3 July 2013

Keywords:
Artefact
Downward continuation
Ellipsoid
Error propagation
Signal and error degree variances
Sphere

A B S T R A C T

So far the recent Earth's gravity model, EGM08, has been successfully applied for different geophysical and geodetic purposes. In this paper, we show that the computation of geoid and gravity anomaly on the reference ellipsoid is of essential importance but error propagation of EGM08 on this surface is not successful due to downward continuation of the errors. Also we illustrate that some artefacts appear in the computed geoid and gravity anomaly to lower degree and order than 2190. This means that the role of higher degree harmonics than 2160 is to remove these artefacts from the results. Consequently, EGM08 must be always used to degree and order 2190 to avoid the numerical problems.

1. Introduction

The recent combined Earth gravity model (EGM), EGM08 (Pavlis et al., 2008), contains spherical harmonics of the gravity field to degree and order 2160, and partially to 2190, corresponding to a spatial resolution of $5' \times 5'$. The long wavelength harmonics of this EGM were extracted from the recent product of the gravity field and climate experiment (GRACE) (Tapley et al., 2005), ITG-GRACE03S (see e.g. Mayer-Guerr et al., 2010) to degree and order 180. Also the satellite altimetry data over the oceans and seas together with the terrestrial gravimetric data were used to recover the harmonics to medium frequencies and the high frequencies were recovered from topographic heights. Today, EGM08 can be used for various purposes including geoid determination due to its relatively high frequencies and its good fit to the gravity fields of different areas all over the globe, except for those places that the topographic and gravimetric data have not good coverage like Antarctica.

EGM08 was used by Eshagh (2009a) to generate the signal spectra of gravity anomaly and compared to the Tscherning/Rapp (Tscherning and Rapp, 1974) model for modifying the Stokes formula. He concluded that the use of EGM08 is beneficial just due to its small error spectra and the result of the modification of the Stokes formula does not differ with the case of using Tscherning/Rapp model. Sjöberg and Bagherbandi (2011a) used this model for studying the crustal structure. Sjöberg and Bagherbandi (2011b) also studied the analytical continuation error of EGM08 and compared it with the topographic bias (Sjöberg, 2007) and concluded that the analytical continuation error is the same with the topographic bias. In another study, Sjöberg (2011) presented a method for estimating the local truncation bias of EGM08 from the terrestrial gravity anomalies and finally derived a very simple formula for that. Sjöberg and Bagherbandi (2012) used EGM08 for the determination of quasigeoid-to-geoid separation. Similar study was done by Bagherbandi and Tenzer (2013) in Himalayas, Tibet and central Siberia using GOCO02S gravity model. Eshagh (2013) calibrated the recent gravity models of the gravity field and steady-state ocean circulation explorer (GOCE) (ESA, 1999) based on EGM08. Since EGM08 is limited to degree and order 2160, therefore, for local geoid determination purpose its truncation bias should be considered. Sjöberg (2011) modelled this bias and Eshagh (2012) used the model and presented an estimator for geoid computation purpose.

The main idea of this paper is to compute an EGM08-based geoid model for Fennoscandia based on an ellipsoidal reference surface and discuss its difference with respect to the spherical one. For example, the standard deviation (STD) of differences between the geoid and GPS/levelling data over Sweden should have been about 28 mm based on the reference ellipsoid while the results of Eshagh (2012) showed that it was 40 cm based on spherical reference. Here, we present the reasons for obtaining such large values and discuss about some practical issues related to the use of EGM08 and the estimator presented by Eshagh (2012). It seems that the main difference is due to computing the geoid and gravity anomaly on the surface of sphere and ellipsoid; and we will show how different the results are on these two surfaces. Another issue is related to propagation of the errors of the EGM08 to geoid and gravity anomaly which we will discuss in the

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0926-9851/$ – see front matter
http://dx.doi.org/10.1016/j.jappgeo.2013.06.011

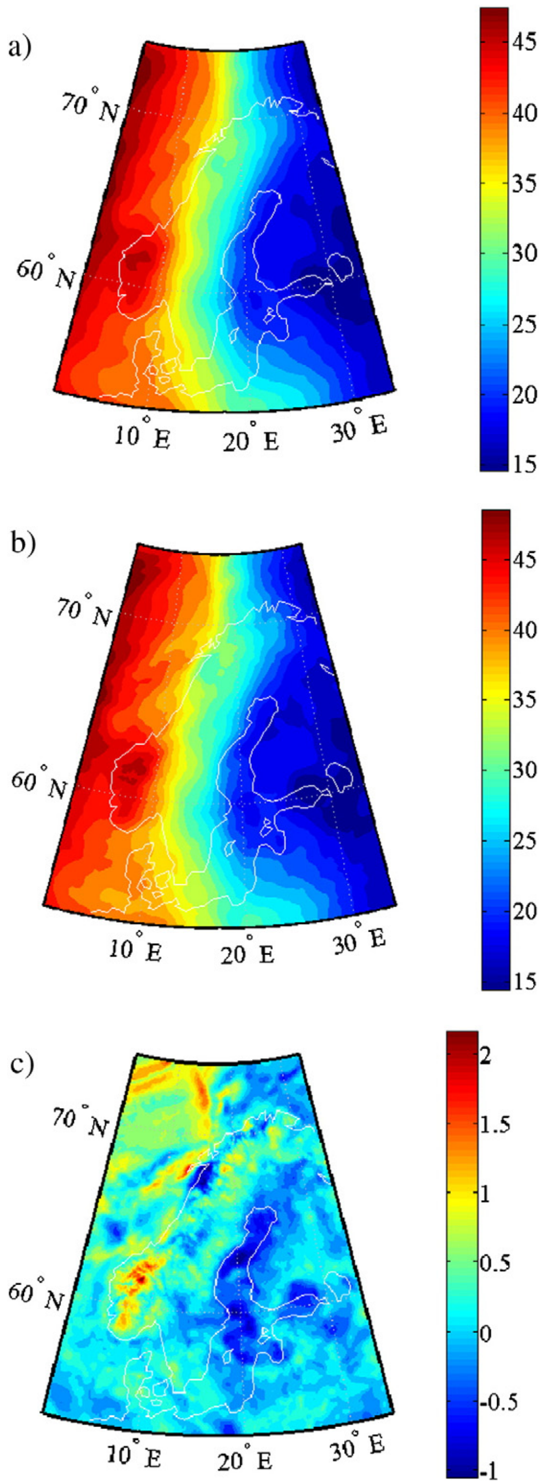

Fig. 1. Geoid model computed by EGM08 to degree and order 2190 computed on a) sphere and b) ellipsoid and c) their differences, unit: 1 m.

paper. Here we numerically investigate generation of geoid and gravity anomaly for a limited geographical region as a case study in Fennoscandia.

2. Spherical harmonic expansion of geoid and gravity anomaly and their errors

An EGM based geoid estimator has been presented by Eshagh (2012) which considers the total topographic effect N_t^T, the total atmospheric effect $\bar{N}_t^A$ computed to degree N_{max} and δN_t^A is the contribution of its higher degrees recovered from the topographic height and an atmospheric density model. This geoid estimator is (Eshagh, 2012):

$$\tilde{N} = \hat{a}\left(N^E + \delta N^E + N_t^T + \bar{N}_t^A + \delta N_t^A\right), \tag{1}$$

where N^E is the geoid model computed using an EGM limited to a specific degree of N_{max} and

$$\hat{a} = \frac{N^2}{N^2 + \theta^2}, \tag{2}$$

where

$$\theta^2 = \left[\sigma^2_{N^E} + k^2\left(\sigma^2_{\Delta g^T} + \sigma^2_{\Delta g^E} - 2\sigma_{\Delta g^T \Delta g^E}\right) + 2k\left(\sigma_{N^E \Delta g^T} - \sigma_{N^E \Delta g^E}\right)\right], \tag{3}$$

where $\sigma^2_{N^E}$ is the variance of the computed geoid by the EGM, $\sigma^2_{\Delta g^T}$ is the variance of the terrestrial gravimetric data, $\sigma_{\Delta g^T \Delta g^E}$, $\sigma_{N^E \Delta g^T}$ and $\sigma_{N^E \Delta g^E}$ are the covariances between the terrestrial and EGM gravity anomalies, the EGM geoid and terrestrial anomaly and the EGM geoid and gravity anomaly, respectively. $\hat{a}$ filters the geoid based on the errors of the data which were used for estimation of the geoid.

The spherical harmonic expansion of geoid is:

$$N^E(P) = \frac{GM}{R\gamma}\sum_{n=2}^{N_{max}}\left(\frac{R}{r}\right)^{n+1}\sum_{m=-n}^{n} t_{nm}\bar{Y}_{nm}(P) \quad r > R \tag{4}$$

where GM is the geocentric gravitational constant, R is the semi-major axis of the reference ellipsoid, r is the geocentric distance of point P, t_{nm} is the spherical harmonic coefficient with degree n and order m, $\bar{Y}_{nm}(P)$ stands for the fully-normalised spherical harmonics and γ is the normal gravity at the surface of the reference ellipsoid.

The spherical harmonic expansion of gravity anomaly is (cf. Eshagh, 2009b):

$$\Delta g^E(P) = \frac{GM}{R^2}\sum_{n=2}^{N_{max}}(n-1)\left(\frac{R}{r}\right)^{n+2}\sum_{m=-n}^{n} t_{nm}\bar{Y}_{nm}(P). \tag{5}$$

According to the propagation law of random errors the errors of the geoid and gravity anomaly, respectively, are:

$$e_N^E(P) = \frac{GM}{R\gamma}\sqrt{\sum_{n=2}^{\infty}\left(\frac{R}{r}\right)^{2n+2}\sum_{m=-n}^{n} e^2_{t_{nm}}\left[\bar{Y}_{nm}(P)\right]^2} \tag{6}$$

and

$$e_{\Delta g}^E(P) = \frac{GM}{R^2}\sqrt{\sum_{n=2}^{\infty}(n-1)^2\left(\frac{R}{r}\right)^{2n+4}\sum_{m=-n}^{n} e^2_{t_{nm}}\left[\bar{Y}_{nm}(P)\right]^2} \tag{7}$$

where $e_{t_{nm}}$ stands for the error of spherical harmonic coefficients.

3. Discussion and numerical issues

For our numerical test we select Fennoscandia, limited between latitudes 55° N and 70° N and the longitudes 0° E and 35° E. This area contains about 4400 GPS/levelling and 1 million gravimetric data with good quality. Here, we want to show the complexity of generating the geoid and gravity anomaly and their errors on the surface of the reference ellipsoid. We compare our results with the terrestrial gravimetric and GPS/levelling data over Fennoscandia. Here, we just focus on the generation of N^E and Δg^E and their errors by EGM08 for the rest of the parameters presented in Eq. (1) see Eshagh (2012) and/or Eshagh and Ebadi (2013).