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Laplace-domain full waveform inversion using irregular finite elements for complex foothill environments

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ABSTRACT

Refraction-traveltime tomography is the most common approach and widely used for estimating velocity models with rugged topography and strongly variant near-surface geology. However, for complex geographical structures, there is often a restriction to the application of the conventional approach because the refracted energy can be trapped by the near-surface structure, which leads to limited depth penetration. To solve this problem, we propose a velocity estimation algorithm for foothill areas using Laplace-domain full waveform inversion (FWI) with irregular finite elements. Because the Laplace-domain FWI uses wavefields damped exponentially in time, the acoustic wave equation can be applied to foothill datasets without suppressing various types of elastic noise. In this study, irregular finite elements are generated to depict complicated surface topography using a Delaunay triangulation and tetrahedralization algorithm. Furthermore, adaptive mesh generation that formulates larger size elements with greater depth is used for minimizing the intensive computational costs in solving the full wave equation in the 2D and 3D domains. The validity of our proposed algorithm is demonstrated for 2D and 3D synthetic datasets and a 2D real exploration dataset acquired in the complex Aquio field foothill area in Bolivia.

1. Introduction

Due to the large number of oilfields in folded and thrust belt areas, these areas have been seismically imaged by many oil and gas exploration companies. Prestack depth migration results are determined by the accuracy of the used velocity model, and refraction-traveltime tomography, using first-arrival traveltimes as input (Stefani, 1995; Zhu and McMechan, 1989), is the most commonly applied velocity estimation method. However, in foothill areas, the conventional method, which uses the first-arrival traveltime of head waves or diving waves, cannot provide correct large-scale velocity information because the refraction energy can be easily trapped by complex near-surface structures (Boonyasiriwat et al., 2009).

Laplace-domain full waveform inversion (FWI), as a macro-velocity estimation technique, was developed to solve the nonlinearity of the inverse problem by utilizing the zero-frequency components of damped wavefields (Shin and Cha, 2008). By adopting the logarithmic misfit function, Laplace-domain FWI can match the weighted amplitudes of both the first and later arrivals to the recorded data, and this increases sensitivity to deeper velocity anomalies compared with refraction tomography (Bae et al., 2012). With the conventional Laplace-domain inversion algorithm, wave modeling based on structured regular mesh has been widely applied due to its computational efficiency. The mesh generation process for a regular mesh is much simpler than it is for an irregular meshing algorithm. In addition, the element node connectivity information does not have to be stored explicitly, and the simple form of the global matrix reduces the computing costs (Pelties et al., 2010). However, using a uniformly structured mesh is problematic when describing elastic domains having complex topographic features through staircase approximation. Subsequently, the wave propagation in an elastic half-space, such as diffractions of incident P, SV and Rayleigh waves (Sánchez-Sesma and Campillo, 1993), can be severely distorted, and the distortion in the wave modeling hinders the successful FWI. To alleviate these problems, irregular meshing is necessary for elastic domains having irregular free surfaces. Recently, there have been several studies that have discussed large-scale elastic wave modeling algorithm using different types of finite element methods (FEMs) with irregular unstructured meshes that have taken advantage of the rapid development and availability in computing resources (Bao et al., 1998; Lee et al., 2008).

The Laplace-domain wavefields are identical to the zero-frequency components of damped wavefields in time; therefore the shape and size restrictions of the finite elements are not as limited by numerical dispersion and spurious reflections as they are for time- and frequency-domain wave modeling (Shin and Cha, 2008). Therefore, we apply the standard FEM using linear irregular elements to the Laplace-domain FWI instead of the high-order finite element approach. In addition, the characteristics of unstructured mesh enable adaptive meshing, which generates smaller elements for the complex topography and larger

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elements for the deep areas. Consequently, the computational burden can be reduced further, which addresses a serious problem in 3D numerical modeling.

In this paper, we briefly introduce the acoustic Laplace-domain FWI algorithm and the feasibility of elastic dataset application. In addition, the superiority of the Laplace domain with respect to stable numerical modeling using irregular finite elements is discussed. We compare the numerical Green's function in a homogeneous background medium with the analytical Green's function to examine the numerical stability of the modeling algorithm using irregular finite elements. Subsequently, the computing performances using adaptive and non-adaptive mesh generation are quantitatively compared. Finally, we apply the FWI algorithm to 2D and 3D synthetic datasets and a real 2D dataset acquired in the geologically complex foothill area of the Aquio field in Bolivia.

2. Theory

2.1. Laplace waveform inversion algorithm

Shin and Cha (2008) proposed that the objective function using the logarithmic wavefields could be effectively applied to a Laplace-domain waveform inversion. For a single Laplace damping constant, the objective function using the log-norm can be expressed as follows:

$$E(\mathbf{m}) = \sum_{i=1}^{n_s} \sum_{j=1}^{n_r} \left\{ ln\left(u_{ij}(\mathbf{m})\right) - ln\left(d_{ij}\right) \right\}^2 \tag{1}$$

where $\mathbf{m}$ is the model parameter (e.g., the velocity, density, or impedance); $u_{ij}(\mathbf{m})$ is the Laplace-domain modeled data (subscripts i and j are the source and receiver numbers, respectively); d_{ij} is the Laplace-transformed data (which can be obtained by the integration of the damped wavefield for a given damping constant); and n_s and n_r are the numbers of shots and receivers, respectively.

The steepest descent direction is required to minimize the objective function, which can be established by taking the partial derivative of the objective function with respect to the model parameter m_k:

$$\nabla_{m_k} E(\mathbf{m}) = \Re\left[\sum_{i=1}^{n_s}\sum_{j=1}^{n_r} \frac{\partial u_{ij}(\mathbf{m})}{\partial m_k}\left(\frac{ln\left(u_{ij}(\mathbf{m})/d_{ij}\right)}{u_{ij}(\mathbf{m})}\right)^*\right], \tag{2}$$

where $\Re$ indicates the real component of the complex value.

By applying a back-propagation algorithm, the steepest descent direction for all elements can be obtained as follows (Shin and Cha, 2008):

$$\nabla E = \Re\left[\sum_{i=1}^{n_s} \left(\mathbf{F}_i^*\right)^t \mathbf{S}^{-1} \mathbf{r}_i^*\right], \tag{3}$$

where $\mathbf{F}_i^*$ is the virtual source matrix (the superscript * denotes the complex conjugate and t represents the transpose operator); $\mathbf{S}^{-1}$ is the inverse of the complex impedance matrix originating from the FEM; and $\mathbf{r}_i$ is the residual wavefield.

In this study, the amplitude of the steepest descent direction is scaled using the diagonal of the pseudo-Hessian matrix proposed by Shin et al. (2001) and the damping factor λ is added to stabilize the descent direction:

$$\nabla E_{scaled} = \left[diag\left(\sum_{i=1}^{n_s} \left(\mathbf{F}_i^*\right)^t \mathbf{F}_i^*\right) + \lambda \mathbf{I}\right]^{-1} \nabla E. \tag{4}$$

After this calibration, the descent direction for each Laplace constant is normalized by its maximum absolute value, which provides the same weighting to the steepest descent directions obtained using each Laplace constant. Subsequently, the final descent direction can be established by summing the descent directions.

Ha et al. (2010) proposed that the Laplace-domain FWI based on acoustic wave equation can be efficiently applied to the elastic datasets from land exploration fields. This approach is inherited from the characteristics of the Laplace-domain wavefield. The Laplace-domain wavefield corresponds to a zero-frequency component of an exponentially damped wavefield in the time domain (Shin and Cha, 2008). Therefore, the various elastic waves traveling slower than the P-wave velocity can be damped out by taking the Laplace transform with several damping constants, rendering their effect insignificant in the acoustic FWI algorithm. Fig. 1a and b compares the time-domain traces with and without elastic noise. Through the damping process, both the elastic noises are completely damped out, and the difference between the damped waves with and without elastic noises disappears (Fig. 1c and d). Using this property in Laplace-domain wavefields, we perform an acoustic Laplace-domain FWI using an elastic dataset.

2.2. Using irregular finite elements in the Laplace domain

Uniformly structured meshes using quadrilaterals or hexahedrons have been widely employed in wave modeling and FWI for several reasons: 1) the regular mesh generation process is simpler and faster than irregular meshing, 2) less memory space is required because the element node connectivity information does not have to be stored explicitly, and 3) the simple form of the global matrix and data structure reduce the computing costs (Pelties et al., 2010). However, the staircase approximation using structured regular mesh cannot precisely describe the irregular topography in foothills, and elastic waves on the free surface can be severely distorted. The advantage of arbitrary triangular elements in approximating to any boundary configuration has been demonstrated by Zienkiewicz et al. (2005). Tetrahedrons also have similar properties to the triangles. These facts motivate us to use triangular and tetrahedral elements for Laplace-domain wave modeling and FWI in the foothills.

In generating the unstructured irregular mesh for the Laplace-domain wave modeling and FWI, we should first define the boundary of the elastic domain using the source and receiver elevation information and the maximum depth of the model. Irregular meshing considering internal discontinuities is not necessary in our study because there is no distinct velocity contrasts in the Laplace-domain inverted model. After defining the domain boundary, we generate the irregular elements using Delaunay triangulation and tetrahedralization algorithms (Shewchuk, 1998). In the FEM, the poorly shaped elements mainly characterized by the small angle may produce numerical instability in the solution due to the round-off error in floating-point operations (Shewchuk, 2002b). To minimize the numerical instability of the Laplace-domain wavefields, the quality of the elements is controlled. The minimum permitted angle measures the quality of the 2D triangular elements, and it can be controlled by the Delaunay refinement algorithm (Shewchuk, 2002a). If the minimum permitted angle is α, it guarantees that every angle of the whole triangular element is between α and $180 - 2 \times \alpha$ degrees. In the 3D case, the radius-edge ratio, proposed by Miller et al. (1995), is used to measure the quality of the 3D tetrahedral elements. The radius-edge ratio (Q) is defined as $Q = R/L$ where R is the radius of the unique circumsphere of the tetrahedron and L is the length of the shortest edge. Generally, tetrahedrons of higher quality possess a smaller value of the radius-edge ratio. The minimum permitted angles of the regular triangle and the radius-edge ratio of the regular tetrahedron are 60° and 0.612, respectively. As the elements created by a mesh generator become similar to the regular triangle and regular tetrahedron in the 2D and 3D cases, the quality of the mesh is improved, but higher numbers of elements are generated. On the surface, the sizes of the triangles and tetrahedrons should be small enough to precisely approximate the irregular topography. However, in the deeper area, the size of the