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The importance of including conductivity and dielectric permittivity information when processing low-frequency GPR and high-frequency EMI data sets

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ABSTRACT

The full solution for the wavenumber equation in electromagnetic (EM) theory is available, but not routinely used when processing ground penetrating radar (GPR), and electromagnetic induction (EMI) data. The wavenumber approach is important as it is used for the development of concepts such as skin depth and phase velocity, as well as being the basis for more complete interpretation of EM data sets. Approximations that make the solution simpler are common, and sufficiently accurate, provided that the underlying assumptions are not grossly violated. With the advent of lower-frequency GPR systems (25 MHz and below) and higher-frequency EMI systems (greater than 100 kHz) such approximations need to be re-examined. This paper reviews the full wavenumber expression and then compares phase velocity and skin depth equations based on approximations with the equations for the same parameter based on the full solution. This comparison allows the conditions under which the assumptions are valid to be refined. In this paper it is shown that for GPR surveys conducted under transition band conditions, the error in phase velocity estimates based on low-loss assumptions may be 40%. Similarly, for EMI surveys the skin depth estimation errors may be more than 30% when the equation based on quasi-static assumptions is used instead of the full solution.

1. Introduction

GPR surveys conducted in many sedimentary environments are not low-loss and velocities calculated assuming low-loss conditions are not valid. Giroux and Chouteau (2010) discuss limitations in the use of the low-loss approximation under lossy conditions when calculating dielectric permittivity and water content derived from GPR data. Similarly, Huang and Fraser (2001, 2002) and Yin and Hodges (2005) recognize that EMI data collected using airborne systems in resistive terranes are potentially affected by variations in dielectric permittivity, and that the assumption that conditions are quasi-static is not adequate. Kalscheuer et al. (2008) show that under relatively resistive conditions the inclusion of dielectric permittivity information when processing radiomagnetotelluric data improves the results significantly.

In this paper we look at how solutions to the wavenumber equation are used to estimate phase velocity and skin depth for simple EMI and GPR problems, and define ground conditions under which the low-loss and quasi-static equations may be used. Using this approach we define the transition band between the two methods where results from each may be improved by using full-solution formulations. The consequences of deriving information about the near surface are considered using hypothetical field examples, comparing calculated parameters based on the approximations and the full solution. Additionally a 2 km long section of GPR data is processed twice: first using simple assumptions of low-loss conditions; and then using the equations shown in this paper. Comparison of these results suggests that conversion of time data to depths under transition band conditions is improved when the correct equation is used, incorporating conductivity information when estimating phase velocities.

2. Theoretical background

The solution for electric field strength, using Maxwell's equations and the constitutive relations, assuming plane wave, sinusoidal time-dependence in a homogeneous isotropic earth (e.g. Ward and Hohmann (1991)) is:

$$E = E_0 e^{-i(kz-\omega t)} \quad (1)$$

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where E is the electric field strength vector (V/m), E_0 is the initial electric field magnitude, z is depth (m), t is time (s), and ω is angular frequency (given by $2\pi f$, where f is frequency). The wavenumber, k, is defined as:

$$k = \left(\omega^2 \varepsilon\mu - i\omega\mu\sigma\right)^{\frac{1}{2}} = \beta - i\alpha \tag{2}$$

where μ is magnetic permeability (H/m), σ is electrical conductivity (S/m), and ε is dielectric permittivity (F/m). We note that ε is often expressed as $\varepsilon_0\varepsilon_r$, where ε_0 is the permittivity of free space (8.854×10^{-12} F/m) and ε_r is the (dimensionless) relative dielectric permittivity. Similarly, μ is often expressed as $\mu_0\mu_r$, where μ_0 is the magnetic permeability of free space ($4\pi \times 10^{-7}$ H/m) and μ_r is the (also dimensionless) relative magnetic permeability. As μ_r is very close to one for many near-surface materials (Lazaro-Mancilla and Gomez-Trevino, 2000), we assumed that to be the case for this paper.

The terms α and β are the attenuation and phase terms (von Hippel, 1954) that need to be determined. Stratton (1941) gives the following solutions:

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left\{\sqrt{\left(1+\frac{\sigma^2}{\varepsilon^2\omega^2}\right)}-1\right\}} \tag{3}$$

and:

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left\{\sqrt{\left(1+\frac{\sigma^2}{\varepsilon^2\omega^2}\right)}+1\right\}} \tag{4}$$

From these the complete expressions for phase velocity and skin depth are given by:

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\frac{\mu\varepsilon}{2}\left\{\sqrt{\left(1+\frac{\sigma^2}{\varepsilon^2\omega^2}\right)}+1\right\}}} \tag{5}$$

$$\delta = \frac{1}{\alpha} = \frac{1}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left\{\sqrt{\left(1+\frac{\sigma^2}{\varepsilon^2\omega^2}\right)}-1\right\}}} \tag{6}$$

The loss tangent is defined as:

$$\tan\theta = \frac{\sigma}{\varepsilon\omega} \tag{7}$$

Usually, simplifying assumptions are made to solve for α and β in Eq. (2). For GPR, under low-loss conditions, when the loss tangent is $\ll 1$, it is assumed that conduction currents are small, and therefore the $i\omega\mu\sigma$ term in Eq. (2) is ignored (Annan, 2005; Ward and Hohmann, 1991). In this case, $\alpha \cong 0$ (i.e. that there is very little attenuation) and $\beta \cong \omega\sqrt{\mu\varepsilon}$. Phase velocity in GPR, under low-loss conditions is defined as:

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} \tag{8}$$

Similarly for EMI methods, Eq. (2) is simplified by assuming that displacement currents are small, the loss tangent is $\gg 1$, and therefore the $\omega^2\varepsilon\mu$ term may be ignored (Ward and Hohmann, 1991). In this case $\alpha = \beta = \left(\frac{\omega\mu\sigma}{2}\right)^{\frac{1}{2}}$. From here, the standard definition of skin depth is given as:

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}} \tag{9}$$

The phase velocity as defined under EMI conditions (Ward and Hohmann, 1991) is given as:

$$v = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}} \tag{10}$$

As the frequency bandwidth of GPR and EMI techniques is extended there are situations where the system is operating in the transition band between low-loss and quasi-static conditions, when the loss tangent is no longer $\ll 1$ for the low-loss case, or $\gg 1$ for the quasi-static case. It then becomes necessary to use the more complete expressions for phase velocity and skin depth given in Eqs. (5) and (6). In examining these equations it is apparent that knowledge about both the conductivity structure and dielectric permittivity structure of a survey area are needed to fully solve for phase velocity and skin depth.

2.1. Dispersion

Dispersion is the variation of velocity with frequency, resulting in the distortion of the shape of a wavetrain as it passes through a medium (Sheriff, 2002). In general, dispersion complicates the interpretation of both GPR and EMI methods. Annan (1996) states that there are four main mechanisms which cause dispersion in GPR. These are transmission dispersion, physical property dispersion, reflection dispersion and scattering dispersion. The first two, transmission and physical property dispersion are important influences to EMI methods as well, and will be discussed in more detail here.

Transmission dispersion affects the conduction part of the conduction-to-displacement current continuum of EM field propagation. Fig. 1 (after Annan, 1996) illustrates the mechanism behind transmission dispersion. The solid line shows how velocity varies with frequency, under a specific set of ground conditions, based on Eq. (5). The lines

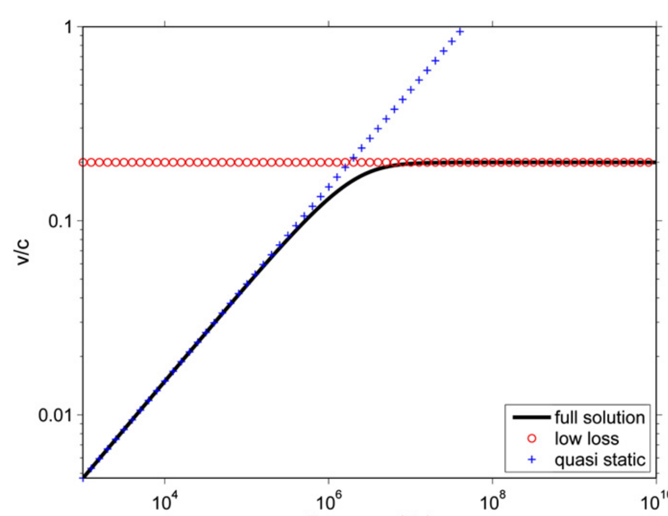

Fig. 1. Phase velocity variation with frequency. Relative dielectric permittivity = 25, conductivity = 5 mS/m. The solid line shows variation using the full solution (Eq. (5)); the line marked with circles shows variation using the low-loss approximation (Eq. (8)); the line marked with crosses shows variation using the quasi-static approximation (Eq. (10)). Modified from Annan (1996).