Journal of Applied Geophysics 95 (2013) 104–114

Contents lists available at SciVerse ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

Computing simple models for scatterers in eddy current problems using a modal decomposition

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ARTICLE INFO

Article history:
Received 29 November 2012
Accepted 16 May 2013
Available online 24 May 2013

Keywords:
Electromagnetic induction
Discrete spectrum of relaxations
Eddy current modeling
Finite elements
Body of revolution
Singularity expansion method

ABSTRACT

A modal decomposition method for computing the solution to an eddy-current problem is presented. Modes are computed using the finite element method and the modes produced are simplified to a magnetic polarizability dyadic in the form of a singularity expansion. This reduced form is a very compact and easily-implemented model for the scatterer's reaction to an arbitrary magnetic field. Due to scaling properties, a single model can be applied to scatterers with different sizes and conductivities. The modal decomposition method could be used to compute a variety of parameterized simple models for canonical shapes to assist in algorithm design for clutter discrimination in buried object detection systems. The method is verified using analytically known formulas for the dyadic of a sphere and a loop. The code is then used to predict the behavior of cylinders with arbitrary conductivity and size, showing excellent agreement with a set of measured cylinders.

1. Introduction

One of the main difficulties in buried object detection for the purpose of clearing land mines or unexploded ordnance is distinguishing between a target of interest and a clutter target. Electromagnetic induction (EMI) sensors show great promise in this respect, as these sensors are much less responsive to variations in the properties of soils than their electric or higher frequency electromagnetic counterparts. When dealing with a soil that is not itself magnetic, EMI sensors reduce the clutter problem to that of distinguishing between the responses of different metallic objects. However, in regions with large amounts of man-made metallic debris, the clutter problem can still make dangerous target detection with a traditional metal detector too time-consuming to pursue.

A somewhat successful method for distinguishing metallic clutter from desired targets is the use of wideband magnetic excitations. Sensors might excite the buried object using a continuous wave multisine (Scott, 2007) or with a broadband pulse (Nelson et al., 2001). The additional frequency data may then be used to delineate clutter from non-clutter using a manually designed or machine-trained algorithm. To use the additional frequency data obtained from a broadband sensor in an intelligent way, a number of simple models for describing broadband induction have been proposed.

Several of these algorithms are based on Carl Baum's singularity expansion method (Baum, 1999a). In Baum (1999a), it is proposed that the inductively excited target be modeled by its magnetic polarizability dyadic as a function of frequency. Models that attempt to extend this idea to deal with larger targets have also been proposed (Sun et al., 2006; Zhang et al., 2001). In Barrowes et al. (2004), an alternative approach is proposed where a target is approximated by the best-fit prolate spheroid. We will focus on Baum's original magnetic polarizability dyadic formulation in this paper.

A number of papers have used Baum's model to justify the expansion of the frequency response in terms of a singularity expansion of real-valued poles (Tantum et al., 2013; Wei et al., 2009). The interest in this approach is generally motivated by the fact that the relaxation time constants obtained are independent of the target's orientation, which allows one to neglect the spatial behavior of the response. Because of this orientation-independence, the main computational work on the polarizability dyadic has limited the discussion to the calculation of the pole values (Carin et al., 2001; Geng et al., 1999). However, the spatial shape of the target response can be a significant indicator of target symmetry, as discussed in Baum (1999b), and some work focusing specifically on the spatial response has been done using time-domain systems (Smith and Morrison, 2005).

In this work we provide a complete path for computing the magnetic polarizability dyadic. Rather than basing our work on Baum's singularity expansion method, we instead begin with the eigenvalue formulation proposed and later implemented with FEM in Davey and Turner (1987) and Wong and Cendes (1988). This approach is modal in nature, and therefore may be more familiar to engineers than the singularity expansion method. It also has the advantage of providing an eigenvalue system, which gives access to the modes of the scatterer and efficient routines for locating the pole locations.

Because the use of finite-elements in eddy current work is not new, even specifically for use in the EMI problem (Faircloth et al., 2004; Hiptmair, 2002), the majority of this paper is dedicated to the

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http://dx.doi.org/10.1016/j.jappgeo.2013.05.009

modal solution and how the modes are used to construct the magnetic polarizability dyadic. Much of this is independent of the numerical method used to obtain the modes. For simplicity of meshing and viewing, we use the body-of-revolution (BOR) assumption to compute the modes, but the modal method does not require the assumption.

In this paper, we begin with Section 2 by describing the modal solution and how it provides the form of a singularity expansion of the scattered field. Next, we show how the computed modes can be simplified to a term in the singularity expansion of the magnetic polarizability dyadic. In Section 3 we provide a brief description of the BOR-FEM code implemented to obtain the modes. In Section 4, the output of the code is compared with the available analytic magnetic polarizability dyadics and shows excellent agreement. In Section 5, a graph of the response of an arbitrary cylinder is given up to the first two poles and compared with measured data using the polarizability dyadic measurement system described in Scott and Larson (2010).

2. The mode-based eddy current model

In this section, we begin by providing a modal solution to the eddy-current problem. We show that this solution takes the form of a singularity expansion of the scattered field response. After describing how the modes are obtained, we will show how they can be simplified to an approximate representation as a portion of the magnetic polarizability dyadic.

2.1. Field-based model of the eddy current response

Consider a conductive region like that shown in Fig. 1. The total fields in this region, $\left(\vec{\mathbf{E}}^t, \vec{\mathbf{H}}^t\right)$, satisfy Maxwell's equations. We make the standard eddy-current approximation of neglecting the displacement current and apply a two-sided Laplace transform to the equation in the variable, s. The equations are,

$$\nabla \times \vec{\mathbf{E}}^t = -s\mu\vec{\mathbf{H}}^t \tag{1}$$

$$\nabla \times \vec{\mathbf{H}}^t = \sigma\sigma_r\vec{\mathbf{E}}^t \tag{2}$$

$$\nabla \cdot \vec{\mathbf{E}}^t = 0 \tag{3}$$

$$\nabla \cdot \vec{\mathbf{H}}^t = 0, \tag{4}$$

where μ is the permeability in the space and σ is the conductivity. Each of these is assumed to be a constant. The function σ_r takes the value,

$$\sigma_r\left(\vec{\mathbf{r}}\right) = \begin{cases} 1 & \vec{\mathbf{r}} \in V \\ 0 & \vec{\mathbf{r}} \notin V \end{cases} \tag{5}$$

where $\vec{\mathbf{r}}$ is the position vector in the space. Note that σ_r can be made non-constant on V with only a minor modification to the following work.

Combining Eqs. (1) and (2), we obtain a Helmholtz-like equation,

$$\nabla \times \nabla \times \vec{\mathbf{E}}^t = -s\mu\sigma\sigma_r\vec{\mathbf{E}}^t. \tag{6}$$

The solution evaluated at $s = j\omega$ corresponds to the Fourier transform of $\vec{\mathbf{E}}^t$ at the angular frequency, ω.

To formulate a scattering problem, we introduce the incident fields, $\left(\vec{\mathbf{E}}^i, \vec{\mathbf{H}}^i\right)$, as shown in the right frame of Fig. 1. These fields are what would be present in the region if the scatterer was not there and they will also satisfy Maxwell's equations, Eqs. (1)–(4), with $\sigma = 0$. For our purposes, the magnetic part of these fields will be constant with space. However, any magnetostatic field produced by a far-away current would suffice. We assume that given an $\vec{\mathbf{H}}^i$ that has no s-dependence and that satisfies Eq. (2), we can easily find an $\tilde{\mathbf{E}}^i$ that satisfies,[1]

$$\nabla \times \tilde{\mathbf{E}}^i = \vec{\mathbf{H}}^i. \tag{7}$$

Solutions for the particular $\vec{\mathbf{H}}^i$ of interest in this paper will be provided explicitly in Section 2.2. We then propose that the incident electric field will take the form of a product, $\vec{\mathbf{E}}^i\left(\vec{\mathbf{r}}, s\right) = \tilde{\mathbf{E}}^i\left(\vec{\mathbf{r}}\right)e^i(s)$. Inserting this representation for $\vec{\mathbf{E}}^i$ into Eq. (1) and using Eq. (7), we obtain

$$\begin{aligned} \nabla \times \vec{\mathbf{E}}^i &= \nabla \times \tilde{\mathbf{E}}^i e^i(s) = \vec{\mathbf{H}}^i e^i(s) = -s\mu\vec{\mathbf{H}}^i \\ &\Rightarrow e^i(s) = -s\mu \end{aligned}$$

This suggests that

$$\vec{\mathbf{E}}^i\left(\vec{\mathbf{r}}, s\right) = -s\mu\tilde{\mathbf{E}}^i. \tag{8}$$

We define the scattered response, $\left(\vec{\mathbf{E}}^s, \vec{\mathbf{H}}^s\right)$, to be the difference between the total and incident fields. Inserting $\vec{\mathbf{E}}^t = \vec{\mathbf{E}}^i + \vec{\mathbf{E}}^s$ into Eq. (6) gives

$$\nabla \times \nabla \times \left(\vec{\mathbf{E}}^i + \vec{\mathbf{E}}^s\right) = -s\mu\sigma\sigma_r\left(\vec{\mathbf{E}}^i + \vec{\mathbf{E}}^s\right).$$

Noting from Eq. (8) and our assumption on $\vec{\mathbf{H}}^i$, that

$$\nabla \times \nabla \times \vec{\mathbf{E}}^i = -s\mu\nabla \times \vec{\mathbf{H}}^i = 0,$$

we obtain the scattering problem,

$$\nabla \times \nabla \times \vec{\mathbf{E}}^s + s\mu\sigma\sigma_r\vec{\mathbf{E}}^s = -s\mu\sigma\sigma_r\vec{\mathbf{E}}^i. \tag{9}$$

We will attempt to solve Eq. (9) using a modal decomposition method. We consider the generalized eigenvalue problem

$$\nabla \times \nabla \times \tilde{\mathbf{E}}_n\left(\vec{\mathbf{r}}\right) = \lambda_n\tilde{\mathbf{E}}_n\left(\vec{\mathbf{r}}\right) \quad \text{when} \quad \vec{\mathbf{r}} \in V \tag{10a}$$

$$\nabla \times \nabla \times \tilde{\mathbf{E}}_n\left(\vec{\mathbf{r}}\right) = 0 \quad \text{when} \quad \vec{\mathbf{r}} \notin V \tag{10b}$$

$$\nabla \cdot \tilde{\mathbf{E}}_n = 0 \quad \text{when} \quad \vec{\mathbf{r}} \notin V \tag{10c}$$

$$\left|\vec{\mathbf{E}}_n\left(\vec{\mathbf{r}}\right)\right| = O\left(\left|\vec{\mathbf{r}}\right|^{-2}\right) \quad \text{as} \quad \left|\vec{\mathbf{r}}\right| \to \infty. \tag{10d}$$

$$\left|\nabla \times \vec{\mathbf{E}}_n\left(\vec{\mathbf{r}}\right)\right| = O\left(\left|\vec{\mathbf{r}}\right|^{-2}\right) \quad \text{as} \quad \left|\vec{\mathbf{r}}\right| \to \infty. \tag{10e}$$

[1] As a rule, tilded vector fields will represent quantities that are independent of the Laplace variable, s. Following electromagnetics convention, both vector fields and vectors will be denoted with an arrow, with the distinction determined from the context.