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Far field elastodynamic Born scattering revisited

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ABSTRACT

This paper summarizes part of an ongoing feasibility study that investigates the possible use of the full elastic Born approximation in multipole borehole acoustics. As a first step we exclude the fluid-filled borehole with the motivation that one or two wavelengths away from the fluid-filled borehole, radiating borehole mode amplitudes (e.g., Stoneley wave, formation dipole wave, etc.) are small compared to body wave amplitudes (*P*-, *SV*- and *SH*-waves). Consequently, for scatterers one or two wavelengths away from the fluid-filled borehole, it suffices to only consider their interaction with body waves.

In this paper we apply the contrast-source stress-velocity *forward scattering* (integral equation) formulation for solid configurations in its first order (Born-) approximation (De Hoop, 1995) assuming a multipole force source excitation in a zero-offset configuration. To scrutinize the validity of the Born approximation, we consider the simplest type of scatterer, i.e., one characterized by a (Heaviside) step function change in one or more of the contrast (perturbation) parameters and we derive analytic zero-offset formulas for the scattered wave particle velocity and displacement in both the space–frequency and space–time domain, respectively. We assume the scatterer to be located in the far-field. More complicated layered configurations can easily be derived by superposition of the given solution types. Explicit results are given for the *dipole* and *quadrupole* excitation, where the former is allowed to have an arbitrary orientation relative to the scatterer and where the latter one is located in a plane perpendicular to the scatterer. In the time domain it is shown, how the scattered wave field decomposes in a *specular* and *diffuse* wave field (two terms borrowed from 'Optics'), where the former contribution vanishes in the absence of an imaging condition and where the latter is always present. For the dipole case, we subject our results to a sensitivity analysis with respect to the three independent perturbation parameters (i.e., density and two compliance parameters) and we compare these results to a full waveform benchmark code that has implemented the reflectivity method (Kennett, 1983), for a 'horizontally' stratified elastic medium. This allowed us to pinpoint the root cause of the observed (small) differences. As it turns out these deviations could be traced back to the inaccurateness of the first order Born scattering coefficients. An additional confirmation of this fact is provided through a comparison between the zero-offset scattering coefficients and the corresponding *Zoeppritz* reflection coefficients. Most notably, it was found that the *PP* first order scattering coefficient needs a higher than quadratic correction in two of the three independent perturbation parameters, i.e., the two compliance parameters, $\delta\Lambda$ and δM. With respect to the density perturbation parameter ($\delta\rho$) the *PP* scattering coefficient correction is quadratic with respect to this perturbation parameter, as is to be expected for a first order approximation. Moreover, also the *SS* first order scattering coefficient only needs a quadratic correction with respect to its associated perturbation parameters, i.e., $\delta\rho$ and δM.

Finally, we give a brief outline on how to numerically implement the Born approximation (employing arbitrary offsets) in a configuration where a source–receiver pair is moving continuously relative to the 'contrast' (Geology), as is the case in borehole acoustic applications.

1. Introduction

Forward and inverse elastic scattering theory is at the root of many applications in seismic exploration, medical imaging, non-destructive testing, etc. Certainly, in the realm of scattering problems in classical physics (i.e., acoustics, elastodynamics and electromagnetics) the fundamental theory is well established and documented. In this respect, perhaps the most complete formulation highlighting the underlying mathematical structure in great detail was presented by De Hoop (1995). In this paper we will use the contrast-source stress-velocity *forward scattering* (integral equation) formulation for solid configurations,

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which due to De Hoop (1995). This configuration generally consists of a background solid (embedding) with known elastodynamic properties and which contains an elastodynamically penetrable object of bounded support (the scatterer) with elastodynamic properties different from those of the embedding. In this configuration the scatterer is elastodynamically irradiated by given sources located in the embedding, in a subdomain outside the scatterer (Fig. 1). The problem is to determine the *total* elastic wave field in the configuration. In this (non-linear) formulation the total wave field can be viewed as the superposition of an *incident* and *scattered* elastic wave field. The incident elastic wave field is the wave field that would be present in the absence of any 'contrast' in elastodynamic properties between scatterer and embedding. The scattered wave field can be viewed as caused by so called *contrast sources* having the scatterer as bounded support. As it turns out these contrast sources can be identified as the contrast weighted *total* elastic wave field components, stress and particle velocity. As a result, in the absence of any contrast in elastodynamic parameters the scattered wave field quantities vanish identically. In the first order linearized approximation of the complete forward elastic scattering formulation, often referred to as the Born approximation, the total elastic wave field components can be replaced by their incident counterparts. This Born approximation will be the subject of this paper. Although many papers have been published regarding this topic, relatively few of these papers deal with its fundamental aspects, especially when it comes to its *fast* forward (analytic) modeling potential of scattered wave events in homogeneous and isotropic layered elastic media. As to the fundamental aspects of the theory, many papers exist that deal with plane wave Born scattering, especially when considering spherical inclusions (e.g., Gubernatis et al., 1977; Korneev and Johnson, 1993a, 1993b, etc.). As to spherical inclusions, there also exist papers that deal with the scattering of an impinging omnidirectional (spherical) wave field and which, in addition, investigate near and far-field effects (e.g., Eaton, 1999; Gritto et al., 1995).

In this paper we will investigate the interaction of an impinging directional (spherical) wave field with the simplest type of scatterer, i.e., the Heaviside (spatial) step function (a half space) and we will focus on the far-field effects. The problem will be solved analytically for the zero-offset case and we will elucidate how this (elementary) solution can be used to synthesize more complicated layered configurations. We will present these (elementary) solutions for two mutlipole force source excitations, i.e., the dipole and the quadrupole excitation. These excitations are of special interest in acoustic wireline logging (WL) and logging while drilling (LWD), respectively. Subsequently we will subject one of our elementary solutions (the dipole excitation) to a sensitivity analysis, i.e., we will systematically change one of the three independent perturbation (elastodynamic 'contrast') parameters (i.e., two compliance parameters and density) and we will compare our derived scattered wave particle velocity results with those obtained from a benchmark code. This code has implemented the reflectivity method (Kennett, 1983), for a 'horizontally' stratified elastic medium (Wang, 1999). In addition, as part of the verification process, we will also compare our derived first order (Born) scattering coefficients with the Zoeppritz reflection coefficients (Zoeppritz, 1919).

Finally, we give a brief outline on how to numerically implement the Born approximation in a configuration where source and receiver are moving continuously relative to the 'contrast' (Geology), as is the case in borehole acoustics.

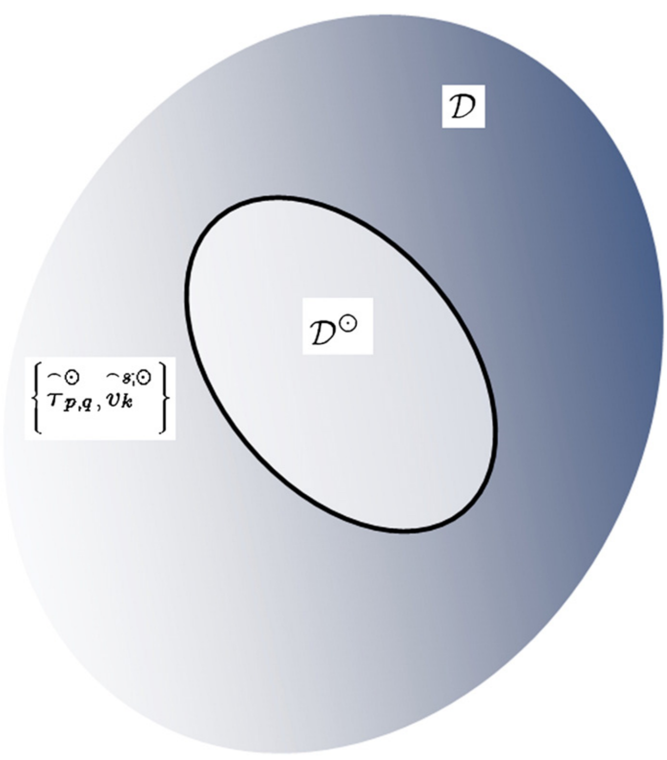

Fig. 1. Schematic view of fundamental domains to be used in the forward source and forward scattering formulation. In the forward source formulation $\mathcal{D}^T$ denotes the spatial support for the elastodynamic (known) sources and we wish to calculate the wave field $\{\tau^T_{p,q}, v_k^{s;T}\}$ everywhere in $\mathcal{D}$. In the forward scattering formulation $\mathcal{D}^{sc.}$ denotes the spatial support for the contrast sources (known in the case of the Born approximation) and we wish to calculate the scattered wave field $\{\tau_{p,q}^{sc.}, v_k^{s;sc.}\}$ everywhere in $\mathcal{D}$.

2. The full elastodynamic forward source and forward scattering formulation

The (full) elastodynamic *forward source* and *forward scattering* formulation for solid configurations can be derived from the global reciprocity theorem of the time-convolution type (De Hoop, 1990, Chapter 15). Both formulations can be summarized by one set of integral equations. In the complex frequency-domain they follow as (De Hoop, 1995, pp. 463–481)

$$\begin{bmatrix} -\hat{\tau}^{\odot}_{i,j} \\ \hat{v}^{s;\odot}_r \end{bmatrix}(\mathbf{x}') = \int_{\mathbf{x}\in\mathcal{D}^{\odot}} \begin{bmatrix} \hat{G}^{\tau,\mathbf{f}^s}_{i,j,k} & \hat{G}^{\tau,h}_{i,j,p,q} \\ \hat{G}^{\mathbf{v}^s,\mathbf{f}^s}_{r,k} & \hat{G}^{\mathbf{v}^s,h}_{r,p,q} \end{bmatrix}(\mathbf{x}',\mathbf{x}) \begin{bmatrix} \hat{f}^{s;\odot}_k \\ \hat{h}^{\odot}_{p,q} \end{bmatrix}(\mathbf{x})dV(\mathbf{x}), \quad \mathbf{x}'\in\mathcal{D}. \tag{1}$$

Next, we will explain the significance of the physical quantities occurring in Eq. (1) in the context of the 'forward source' and 'forward scattering' problem, respectively.

2.1. The forward source formulation

In the *forward source* formulation we wish to express the elastic wave field quantities, $\left\{-\hat{\tau}^{\odot}_{i,j},\ \hat{v}^{s;\odot}_r\right\}(\mathbf{x}')$, $\mathbf{x}'\in\mathcal{D}$ pertaining to a specific solid configuration (Fig. 1) with given elastodynamic properties in terms of the source distributions $\left\{\hat{f}^{s;\odot}_k,\ \hat{h}^{\odot}_{p,q}\right\}(\mathbf{x})$, $\mathbf{x}\in\mathcal{D}^{\odot}$, by which they are generated. In this context $\hat{f}^{s;\odot}_k$ denotes the complex frequency domain solid volume source density of force and $\hat{h}^{\odot}_{p,q}$ denotes the complex frequency domain solid volume source density of deformation rate. For a homogeneous, isotropic, lossless solid the Green's tensors occurring in Eq. (1) are defined as (De Hoop, 1995, Chapter 15, p. 475)

$$\hat{G}^{\tau,\mathbf{f}^s}_{i,j,k}(\mathbf{x}',\mathbf{x}) = (\rho^s)^{-1}C_{i,j,p,q}\partial'_p\,\hat{G}_{q,k}(\mathbf{x}',\mathbf{x}), \tag{2}$$