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Core flows and heat transfer induced by inhomogeneous cooling with sub- and supercritical convection

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The amount and spatial pattern of heat extracted from cores of terrestrial planets is ultimately controlled by the thermal structure of the lower rocky mantle. Using the most common model to tackle this problem, a rapidly rotating and differentially cooled spherical shell containing an incompressible and viscous liquid is numerically investigated. To gain the physical basics, we consider a simple, equatorial symmetric perturbation of the CMB heat flux shaped as a spherical harmonic Y_{11}. The thermodynamic properties of the induced flows mainly depend on the degree of nonlinearity parametrised by a horizontal Rayleigh number $Ra_h = q^* Ra$, where q^* is the relative CMB heat flux anomaly amplitude and Ra is the Rayleigh number which controls radial buoyancy-driven convection. Depending on Ra_h we identify and characterise three distinctive flow regimes through their spatial patterns, heat transport and flow speed scalings: in the linear *conductive* regime the radial inward flow is found to be phase shifted 90° eastwards from the maximal heat flux as predicted by a linear quasi-geostrophic model for rapidly rotating spherical systems. The *advective* regime is characterised by an increased Ra_h where nonlinearities become significant, but is still subcritical to radial convection. There the upwelling is dispersed and the downwelling is compressed by the thermal advection into a spiralling jet-like structure. As Ra_h becomes large enough for the radial convection to set in, the jet remains identifiable on time-average and significantly alters the global heat budget in the *convective* regime. Our results suggest, that the boundary forcing not only introduces a net horizontal heat transport but also suppresses the convection locally to such an extent, that the net Nusselt number is reduced by up to 50%, even though the mean CMB heat flux is conserved. This also implies that a planetary core will remain hotter under a non-homogeneous CMB heat flux and is less well mixed. A broad numerical parameter investigation regarding Rayleigh number and the relative heat flux anomaly further fosters these results.

1. Introduction

The cooling of the liquid iron cores of terrestrial planets is due to radial heat transport towards the core mantle boundary (CMB) via heat conduction and in case the entropy gradient is sufficiently negative supported by buoyancy driven convection. The lateral variation of heat conducted out of the core and hence through the CMB q_{cmb} is mainly controlled by the lower mantle temperature pattern $T_{lm}(\theta, \phi)$, such that

$$q_{cmb}(\theta, \phi) = k \frac{T_{lm}(\theta, \phi) - T_{core}}{\delta_{cmb}}, \quad (1)$$

where k is the thermal conductivity and δ_{cmb} the thickness of the thermal boundary layer at the bottom of the mantle, respectively. To first order, the core temperature is rather uniform due to a much more efficient conductive and convective heat transport therein. If the heat transport is only via conduction, thermal inhomogeneities at the CMB are thought to drive baroclinic flows (Zhang and Gubbins, 1992), whereas in a convecting core lateral variations of convective vigour, the dynamo process and the stimulation of mean horizontal flows are expected.

The importance of thermal coupling between Earth's mantle and core due to inhomogeneities of the lower mantle temperature was first suggested by Bloxham and Gubbins (1987). Seismological evidence for the non-homogeneous CMB heat flux came from the mantle tomography (e.g. Masters et al., 2000) and the detection of LLSVPs (large low shear velocity provinces) (Yuen et al., 1993),

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which are associated with lower local mantle temperatures, but it was also reported that only for an isochemical mantle the variation of shear wave velocity and the temperature is linear (Nakagawa and Tackley, 2008). Such a mantle control has been suggested to influence the geomagnetic secular variation (Bloxham, 2000; Davies et al., 2008; Olson et al., 2010; Aubert et al., 2013) and field strength (Takahashi et al., 2008), concentrate magnetic flux patches (Olson and Christensen, 2002; Amit et al., 2010), lock the (usually drifting) core convection (Davies et al., 2009) and dynamo action (Willis et al., 2007) but also introduce mean large scale flows (Gibbons et al., 2007). Furthermore the CMB heat flux anomalies are also reported to affect the buoyancy flux from the inner core (Aubert et al., 2008; Amit and Choblet, 2009; Gubbins et al., 2011). Note, in contrast to most of these dynamo studies, we focus on the hydrodynamic aspects and hence exclude magnetic fields.

Mantle induced CMB heat flux variations are also expected to influence core flows and the magnetic field generation process of other terrestrial planets. For example the rather localised present-day magnetisation on Mars was targeted to be explained by an equatorial antisymmetric mantle-induced CMB heat flux anomaly of variable strength (Stanley et al., 2008; Amit et al., 2011; Dietrich and Wicht, 2013; Monteux et al., 2015). These studies report the induction of a global magnetic field with strong equatorially asymmetric intensity reminiscent of the recently measured distribution of magnetised crust on the surface of Mars (Acuña et al., 1999). Also the equatorial asymmetry of Mercury's magnetic field (Cao et al., 2014; Wicht and Heyner, 2014) and the axisymmetry of Saturn's magnetic field (Stanley, 2010) were investigated in the framework of a non-homogeneous CMB heat flux.

In addition, terrestrial exoplanets orbiting their host star in close proximity and typically in a synchronous orbit will receive strong stellar irradiation at the near side with a latitudinal maximum at the equator. Most likely, the heating–cooling dichotomy at the surface will result in a smooth azimuthal varying, equatorial symmetric thermal forcing pattern at the CMB reminiscent of which we use here (Y_{11}). There have been attempts to model mantle convection under such a specific heating pattern (Gelman et al., 2011) suggesting the development of a single-plume mantle convection mode. Hence the core flows and the probable induction of a core dynamo in a tidally locked terrestrial exoplanet might be significantly influenced by the enormous difference of stellar irradiation between the near and far side.

The problem of a differentially cooled and rapidly rotating fluid shell has received much attention during the last decades. Zhang and Gubbins (1992) first investigated the pure effect of laterally varying temperature at the outer boundary in the absence of radial convection. The numerical results revealed that the radial inflows are *not* found where the local outer boundary temperature is lowest hence cooling most efficient, but they are phase shifted by a quarter of the azimuthal wavelength of the thermal anomaly to where the *azimuthal gradient* of the temperature is maximal. This results from the vorticity balance between Coriolis and buoyancy terms frequently referred to as a thermal wind balance, but more specifically is a Sverdrup balance in the geophysical fluid dynamics (Pedlosky, 1979, see also below). This implies, that in a rotation dominated system thermal anomalies induce vorticity rather than flows directly. This holds as long as the flows are assumed to be rotation dominated, inviscid and nonlinearities due to temperature advection or inertia are negligible (Gibbons et al., 2007) and is hence not found in models with small rotation rates or infinite Prandtl number (Sun et al., 1994; Zhang and Gubbins, 1996; Davies et al., 2009). A mathematically more straight-forward analysis of the linear quasi-geostrophic model (Busse annulus) by Yoshida and Hamano (1993) including the effect of the magnetic fields, confirmed the azimuthal phase shift when there is no magnetic field altering the leading order force balance.

For a physically more realistic model of the Earth's core, experiments by Sumita and Olson (1999) perturbed the outer boundary of a vigorously convecting and rapidly rotating spherical shell with a local anomalous heat flux. There was also a phase shift between local minimum temperature and position of the inward flow reported, however due to the strongly nonlinear driving it was deformed in a jet-like structure spiralling inwards. As the characteristic hydrodynamic numbers (e.g. Ekman or Reynolds number) in experiments are closer to real planets than those accessible by numerical models, this might reflect better the situation in a realistic planetary core. In a follow-up study (Sumita and Olson, 2002) a broad parameter survey was reported, featuring a detailed description of the flow and scaling relations of how the heat flux anomaly, pattern and strength affects the induced flow amplitudes in azimuthal and radial direction. Typically these experiments are set in a strongly convective regime, where a rather localised and very strong heat flux anomaly induces a sharp front separating the cold east from the hot west and strong azimuthal flows connecting them.

As some of the reported effects are due to complex interactions between core convection, boundary forcing and magnetic fields, a clear physical interpretation might not be always possible. Thus to gather clearer insights we limit this study to the hydrodynamic aspects for the sake of a more coherent physical description. Starting from an analytical formulation of the linear theory, we numerically model core flows induced by thermal CMB inhomogeneities for cores subcritical to buoyancy driven core convection (baroclinic flows) and compare them to models featuring radial convection. Note, also the baroclinic flows obey strong nonlinear features, such as bending or compression of the emerging inward jet. As the various linear and nonlinear flow regimes have been studied only individually, our main focus is to distinguish them and discuss their properties.

More precisely we modify the outer boundary heat flux of rapidly rotating spherical shell with a smoothly varying pattern along azimuthal and latitudinal direction. For simplicity the heating of the shell is exclusively supplied by a constant internal heat source modelling secular cooling of the shell. It was shown that internal heated systems are most sensitive to inhomogeneities at the outer boundary (Hori et al., 2014), as the strongest temperature gradients are typically found close the outer boundary. The specific shape is an anomaly of the outer boundary heat flux of spherical harmonic degree and order one (Y_{11}). This purely equatorially symmetric and non-axisymmetric pattern was taken mainly for application to terrestrial exoplanets orbiting their host star in synchronous rotation. Note, today's Earth CMB heat flux variation is dominated by a sectorial spectral mode Y_{22}, but the core convection is mainly driven from compositional buoyancy release at the inner core boundary. Hence if applied to real planetary systems, our models best describe cores of terrestrial planets before inner core nucleation. Apart from the different azimuthal length scales, our linear results might be applicable to a general sectorial anomaly pattern as shown below on Section 3.1. However, for the nonlinear results the interaction between several sectorial heat flux anomalies are beyond this study. Emphasis is put on a clear physical description of the induced flow structures and the radial (and horizontal) heat transport.

We also vary the amplitude of the CMB heat flux variation q^* relative to the mean value as it controls the strength of the boundary forcing to first order and is not even well known for the Earth's core. We are interested in how sensitive convection is to a thermal boundary anomaly of variable strength. Studying q^* might provide a smooth transition between models with homogeneous and heterogeneous CMB heat flux. Note, we limit q^* to unity to avoid