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Influences of the depth-dependence of thermal conductivity and expansivity on thermal convection with temperature-dependent viscosity

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ABSTRACT

In this paper we carried out numerical experiments of a time-dependent thermal convection in a two-dimensional Cartesian box of aspect ratio (width/height) of 6, in order to study the influences on the convecting flow patterns of the spatial variations in physical properties (viscosity η, thermal conductivity k and expansivity α). A series of calculations by systematically varying the magnitude of spatial variations in these properties showed that the strongly temperature-dependent η induces the change in flow pattern into the "stagnant lid" (ST) regime, regardless of the increase in k and/or decrease in α with depth, where the convection occurs only beneath a stagnant lid of cold fluid at the top surface. In particular, we found that the increase in thermal conductivity k with depth significantly affects the convecting flow patterns in the presence of strong temperature-dependence in η. Compared with those with uniform k, the patterns of ST convection with non-uniform k are characterized by (i) thinner top thermal boundary layers or lids and (ii) larger horizontal length scales of convection cells beneath the stagnant lid. In addition, the variation in k with depth decreases the threshold values of the temperature-dependence in η above which the ST-mode of convection takes place, which may also help stabilize the convection cells of large horizontal length scales beneath stagnant lids. Our results may highlight the potential importance of the increase in thermal conductivity with depth (or pressure) in controlling the planforms of thermal convection in the mantle of terrestrial planets.

1. Introduction

There exist large differences in temperature and pressure within the mantle of terrestrial planets. In the Earth's mantle, for example, a pressure increases by about 135 GPa (Dziewonski and Anderson, 1981) with increasing depth, while a temperature does by about 3500 K (Kawai and Tsuchiya, 2009; Stacey and Davis, 2008). These large differences in temperature and pressure are expected to yield substantial variations in the physical properties of mantle materials and, hence, significantly affect the dynamic behaviors in the mantle. Indeed, as had been inferred by several earlier studies (Forte and Mitrovica, 2001; Karato and Karki, 2001; Roberts and Zhong, 2006; Zhong and Zuber, 2001, for example), the spatial variations in mineral properties play an important role in the internal structures observed in the mantle of the Earth and terrestrial planets. In this paper, we will study how the spatial variations of physical properties of mantle materials affect the flow patterns of thermal convection.

Among the spatial variations in fluid-dynamical properties of mantle materials, the influence of the temperature-dependent viscosity on the pattern of thermal convection has been intensively investigated so far, mainly by using theoretical (Stengel et al., 1982), experimental (Davaille et al., 2002; Gonnermann et al., 2004; Jellinek et al., 2002; Robin et al., 2007), and numerical (Kameyama and Ogawa, 2000; Moresi and Solomatov, 1995; Ogawa et al., 1991; Solomatov, 1995; Solomatov and Moresi, 1997) studies in Cartesian domains. Depending on the magnitude of temperature-dependence of viscosity, three convective regimes have been identified from the changes in vertical flow structures; the small-viscosity-contrast or whole layer (WH) regime where convection involves all of the fluid in the vessel, the stagnant lid (ST) regime where convection occurs only beneath a cold stagnant lid which develops along the surface boundary, and the transitional (TR) regime between the WH and ST regimes. Convection of the WH, TR, and ST modes arises when the temperature-dependence of viscosity is weak, moderate, and strong, respectively, and the transitions between the three convective regimes are induced by the changes in the stiffness of the cold thermal boundary layer coming from the changes in the magnitude of temperature-dependence of viscosity.

On the other hand, it has also been recognized that there are spatial variations in physical properties other than the viscosity in the mantle of terrestrial planets. Recent experimental and

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theoretical studies on mineral physics demonstrated, for example, a moderate increase in the thermal conductivity with increasing depth in the Earth's mantle (Goncharov et al., 2010; Hofmeister, 1999; de Koker, 2009, 2010; Tang and Dong, 2010; Ohta et al., 2012, for example), mainly due to the contribution of lattice vibrations (or phonons) which are strongly affected by pressure. A moderate decrease in the thermal expansivity is also expected (Chopelas and Boehler, 1992; Duffy and Ahrens, 1993; Katsura et al., 2009, 2004; Kono et al., 2010; Mosenfelder et al., 2009, for example), mainly due to the increase in pressure.

Unfortunately, little efforts had been devoted so far to the studies on the effects of pressure-dependence in thermal conductivity and expansivity on convective flow patterns. This is probably because the magnitude of their variation with depth is expected to be much smaller than that of viscosity which may vary by many orders of magnitude throughout the mantle. However, it is not obvious at all if the effects of the variations in these properties can be safely ignored in the studies of mantle dynamics, even though their variations are quite modest. Indeed, some earlier studies (Hansen et al., 1993; van den Berg et al., 2005, 2002; Dubuffet et al., 1999; Ghias and Jarvis, 2008; Tosi et al., 2010, 2013; Yanagawa et al., 2005, for example) demonstrated that, under certain conditions, there occurs a significant change in flow patterns by introducing the pressure-dependence in thermal conductivity and/or expansivity.

Recently we have studied by using linear analysis how the structures of incipient convection flows are affected by the spatial variations in the thermal conductivity k, expansivity α, and viscosity η (Miyauchi et al., accepted for publication). A systematic survey in wide ranges of parameters describing the magnitudes of spatial variations in those parameters has demonstrated that the interplay among the spatial variations in these properties may have strong impacts on the nature of incipient convection. In particular, we have found that the spatial variations in k strongly affect both the vertical and horizontal structures of the convection cells of the ST regime occurring when the viscosity is strongly dependent on temperature. It is not obvious, however, if these results are truly applicable to the cases of actual nonlinear (or a finite-amplitude) thermal convection, since the linear theory employed in the above study is in essence valid only near the critical conditions where a fluid motion is infinitesimally small. It is therefore quite important to conduct a systematic study of finite-amplitude thermal convection with the influence of the spatial variations in physical properties, in order tho deepen the insights into the vertical and horizontal structures of convection in the mantle of terrestrial planets.

In this paper, we carry out numerical experiments of time-dependent thermal convection of a fluid in order to study the nature of convection flow patterns using a wide parameter space of the amplitude of spatial variations in physical properties such as temperature-dependent viscosity, pressure-dependent thermal conductivity and/or expansivity. In particular, we will focus on the influences of these variations on the vertical and horizontal convection flow structures when the viscosity strongly depends on temperature. We will begin with revisiting earlier studies (Kameyama and Ogawa, 2000; Solomatov, 1995; Solomatov and Moresi, 1997) in the absence of the pressure-dependent thermal conductivity and expansivity. By comparing the results with those in the presence of their variations, we will discuss the changes on the vertical and horizontal convection flow patterns, time-averaged quantities, and the convection regimes.

2. Model descriptions

We consider a time-dependent thermal convection of a Newtonian, incompressible and infinite-Prandtl-number fluid with temperature-dependent viscosity, pressure-dependent thermal conductivity and expansivity in a basally heated two-dimensional rectangular box of height L and aspect ratio $\lambda = 6$. Boussinesq approximation is employed and, hence, the effects of adiabatic heating and viscous dissipation are ignored in the energy equation. There is no internal heat source in the box. Temperature is fixed at $T_{\rm top}$ and $T_{\rm bot}$ at the top and bottom boundaries, respectively. The impermeable and shear-stress-free conditions are adopted along the top and bottom boundaries while the periodic boundary condition is applied on the vertical side walls. The initial conditions are chosen to make two convection cells with an ascending flow at $x = \lambda L/2$, where x is the horizontal coordinate $(0 \leqslant x \leqslant \lambda L)$.

The viscosity η of fluid is assumed to exponentially depend on temperature T as,

$$\eta(T) \equiv \eta_{\rm top} \exp\left[-\ln(\gamma_\eta)\frac{T - T_{\rm top}}{T_{\rm bot} - T_{\rm top}}\right], \quad (1)$$

where $\eta_{\rm top}$ and γ_η are the viscosity for $T = T_{\rm top}$ and a dimensionless parameter controlling the temperature-dependence of viscosity, respectively. This equation means that the viscosity $\eta_{\rm bot}$ at the bottom surface $(T = T_{\rm bot})$ is equal to $\eta_{\rm top}/\gamma_\eta$. On the other hand, the thermal conductivity k and expansivity α are linearly dependent on pressure (or depth) as,

$$k(z) \equiv k_{\rm top}\left[1 + (\gamma_k - 1)\left(1 - \frac{z}{L}\right)\right], \quad (2)$$

$$\alpha(z) \equiv \alpha_{\rm top}\left[1 + (\gamma_\alpha - 1)\left(1 - \frac{z}{L}\right)\right], \quad (3)$$

where z is the height from the bottom surface, $k_{\rm top}$ is the thermal conductivity at the top surface $(z = L)$, and $\alpha_{\rm top}$ is the thermal expansivity for $z = L$. The depth-dependence of k and α are controlled by the dimensionless parameters γ_k and γ_α, respectively. Namely, the values of thermal conductivity $k_{\rm bot}$ and expansivity $\alpha_{\rm bot}$ at the bottom surface $(z = 0)$ are given by the $k_{\rm top}\gamma_k$ and $\alpha_{\rm top}\gamma_\alpha$, respectively. For a geophysical significance, we restrict ourselves to the cases where $\gamma_k \geqslant 1$ and $\gamma_\alpha \leqslant 1$: the thermal conductivity k increases with depth, while the thermal expansivity α decreases with depth. We varied these parameters in the ranges of $10^0 \leqslant \gamma_\eta \leqslant 10^6$, $1 \leqslant \gamma_k \leqslant 10$, and $0.1 \leqslant \gamma_\alpha \leqslant 1$. In order to focus on the effects of spatial variations in η, k, and α, the spatial variations in other physical quantities (such as specific heat and gravitational acceleration) are ignored. In addition, the Rayleigh number of thermal convection ${\rm Ra}_{\rm bot}$ is defined by:

$${\rm Ra}_{\rm bot} = \frac{\rho_0^2 C_{p0} g_0 \alpha_{\rm bot}(T_{\rm bot} - T_{\rm top})L^3}{k_{\rm bot}\eta_{\rm bot}}, \quad (4)$$

where ρ_0, g_0, and C_{p0}, are the reference values of density, gravitational acceleration, and specific heat at constant pressure, respectively. Namely, ${\rm Ra}_{\rm bot}$ is the Rayleigh number defined with the physical properties at the bottom boundary $(\eta_{\rm bot}, \kappa_{\rm bot}, \alpha_{\rm bot})$.

The basic equations are discretized by the finite-difference method based on the control-volume method. A uniform mesh is employed with 128 and 384 mesh points in vertical and horizontal directions, respectively. In some cases where a very high heat flow is expected across the top surface, we also calculated with finer mesh points (192 points in vertical direction) in order to resolve thin thermal boundary layers. An iterative solution algorithm (Kameyama et al., 2005) is used to calculate the flow field, where the discretized equations for the conservation of momentum and mass are simultaneously solved for the primitive variables (velocity and dynamic pressure) by the pseudo-compressibility method. The energy equation is discretized by the Euler method in time. An upwind scheme, called power-law scheme (Patankar, 1980), is used to evaluate the advection and diffusion terms in the energy equation. The numerical techniques employed in this study are