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Response surface methods for slope reliability analysis: Review and comparison

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ABSTRACT

This paper reviews previous studies on developments and applications of response surface methods (RSMs) in different slope reliability problems. Based on the review, four types of soil slope reliability analysis problems are identified from the literature, including single-layered soil slope reliability problem ignoring spatial variability, single-layered soil slope reliability problem considering spatial variability, multiple-layered soil slope reliability problem ignoring spatial variability, and multiple-layered soil slope reliability problem considering spatial variability, which are referred to as "Type I–IV problems" in this study. Then, the computational efficiency and accuracy of four commonly-used RSMs (namely single quadratic polynomial-based response surface method (SQRSM), single stochastic response surface method (SSRSM), multiple quadratic polynomial-based response surface method (MQRSM), and multiple stochastic response surface method (MSRSM)) are systematically compared for cohesive and c–ϕ slopes, and their feasibility and validity in the four types of slope reliability problems are discussed. Based on the comparison, some suggestions for selecting relatively appropriate RSMs in slope reliability analysis are provided: (1) SQRSM is suggested as a suitable method for the single-layered soil slope reliability problem ignoring spatial variability (i.e., Type I problem); (2) MQRSM is applicable to the multiple-layered soil slope reliability problem ignoring spatial variability (i.e., Type III problem); and (3) MSRSM is suggested to solve slope reliability problems (including single-layered and multiple-layered slopes) considering spatial variability (i.e., Type II and IV problems).

1. Introduction

Reliability analysis of soil slopes has gained considerable attention in the geotechnical reliability community over the past few decades (e.g., Baecher and Christian, 2003; Low and Tang, 2004; Cho, 2007, 2009, 2010, 2013; Fenton and Griffiths, 2008; Ching et al., 2009; Wang et al., 2010, 2011; Ji and Low, 2012; Ji, 2014; Zhang et al., 2013a,b; Jiang et al., 2014, 2015; Li et al., 2011, 2014, 2015a). Many reliability methods have been proposed for slope reliability analysis in literature, such as the first-order second moment method (FOSM) (e.g., Christian et al., 1994; Hassan and Wolff, 1999; Duncan, 2000; Xue and Gavin, 2007; Suchomel and Mašin, 2010), first-order reliability method (FORM) (e.g., Low and Tang, 1997, 2004; Low, 2007; Cho, 2007; Hong and Roh, 2008; Ji, 2014; Zeng and Jimenez, 2014), second-order reliability method (SORM) (e.g., Cho, 2009; Low, 2014), and Monte Carlo Simulation (MCS) (e.g., El-Ramly et al., 2002, 2005; Griffiths and Fenton, 2004; Hsu and Nelson, 2006; Cho, 2007, 2010; Huang et al., 2010, 2013; Tang et al., 2015; Li et al., 2015c) and its advanced variants (e.g., Ching et al., 2009; Wang et al., 2010, 2011; Li et al., 2015d).

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In addition to the aforementioned reliability methods, response surface methods (RSMs) have been used for slope reliability problems with implicit performance functions (e.g. Wong, 1985; Xu and Low, 2006; Ji and Low, 2012; Zhang et al., 2011b, 2013b; Jiang et al., 2014, 2015; Li et al., 2015a,b; Li and Chu, 2015). RSMs have been proved to be an efficient method for slope reliability analysis. For instance, Wong (1985) applied RSM to evaluate the reliability of a homogeneous slope. Xu and Low (2006) used RSM to approximate the performance function of slope stability in slope reliability analysis, in which the response surface is taken as a bridge between stand-alone numerical packages and spreadsheet-based reliability analysis. Recently, several researchers (e.g., Zhao, 2008; Li et al., 2013; Samui et al., 2013) proposed a support vector machine (SVM)-based RSM to approximate implicit performance function using a small set of actual values of the performance function. Taking the radial basis function neural network (RBFN) as an approximate response surface function for the actual performance function, Tan et al. (2011) discussed similarities and differences between RBFN-based RSMs and SVM-based RSMs, which indicated that there is no significant difference between them. To reduce the number of evaluations of the actual performance function, Tan et al. (2013) proposed two new sampling methods and a hybrid RSM. Similar to SVM-based RSM, relevance vector machine (RVM)-based FOSM is adopted to build a RVM model to predict the implicit performance function and evaluate the partial derivatives with sufficient accuracy (Samui et al., 2011). In addition, the artificial neural network (ANN) technique (e.g., Cho, 2009; Chen et al., 2011), the Gaussian process regression (Kang et al., 2015), Artificial bee colony (ABC) algorithm optimized support vector regression (SVR) (Kang and Li, 2015), the high dimensional model representation (HDMR) (Chowdhury and Rao, 2010) and neural networks (NN)-based RSM (Piliounis and Lagaros, 2014) can be also used to establish a relationship between the factor of safety and soil parameters. Luo et al. (2012a,b) and Zhang et al. (2011a) adopted Kriging-based response surface to simulate performance functions and demonstrated its applications in solving several geotechnical reliability problems (Zhang et al., 2013a). Yi et al. (2015) indicated that the particle swarm optimization–Kriging model has a good curve fitting performance.

Recently, an important advance in slope reliability analysis using RSM is that multiple response surface methods were proposed to evaluate system reliability of slope stability. For example, Zhang et al. (2011b) first proposed the multiple response surfaces method for slope reliability analysis. Ji and Low (2012) constructed a group of stratified response surfaces corresponding to the most probable failure modes, and slope system reliability is evaluated based on these stratified response surfaces. Using the quadratic polynomial-based response surface method, Zhang et al. (2013b) extended the Hassan and Wolff method into a practical tool for system reliability analysis.

The aforementioned studies have not considered the spatial variability when the RSMs are used to evaluate slope reliability problems. Ji et al. (2012) made a first attempt to solve slope reliability with spatial variability by the RSM with second-order polynomial approximate function without cross terms. Based on the stochastic response surface method (SRSM), Jiang et al. (2014) proposed a non-intrusive stochastic finite element method for slope reliability analysis considering spatial variability in shear strength parameters, by which the system reliability of soil slopes considering spatial variability is evaluated by the multiple stochastic response surface method (e.g., Jiang et al., 2015; Li et al., 2015a; Li and Chu, 2015).

Based on the above studies, it can be seen that significant advances have been made in applications of RSMs in soil slope reliability analysis. Essentially, the RSM uses a computationally efficient model to approximate the original analysis model (e.g., limit equilibrium analysis or finite element analysis). Then, slope reliability analysis is carried out based on the explicit performance function represented by the RSM. Table 1 summarizes the applications of RSM-based reliability methods in soil slope reliability analyses. These references are listed in a chronological order. The soil slope reliability problems concerned in the previous studies on RSMs can be divided into four categories according to the probabilistic model of soil properties (e.g., random variable or random field models) and slope types (e.g., single-layered or multiple-layered): (1) single-layered soil slope reliability problem ignoring spatial variability (i.e., Type I problem); (2) single-layered soil slope reliability problem considering spatial variability (i.e., Type II problem); (3) multiple-layered soil slope reliability problem ignoring spatial

Table 1
Summary of applications of RSMs in soil slope reliability analyses.

Paper ID	Authors	Year	Types of response surfaces	Single response surface	Multiple response surfaces	Spatial variability		Slope type		Deterministic slope stability analysis
						No	Yes	Single-layered	Multiple-layered	
1	Wong	1985	Quadratic polynomial	√		√		√		FEM
2	Xu and Low	2006	Quadratic polynomial without cross terms	√		√			√	LEM (Spencer), FEM
3	Zhao	2008	SVM-based response surface	√		√		√	√	LEM (Simplified Bishop, Spencer)
4	Cho	2009	ANN-based response surface	√		√			√	FDM
5	Chowdhury and Rao	2010	High dimensional model representation	√		√			√	LEM (Simplified Bishop, Janbu, Morgenstern–Price, Spencer, GLE)
6	Chen et al.	2011	SVM-based response surface	√		√			√	LEM (Morgenstern–Price)
7	Tan et al.	2011	RBFN and SVM-based response surface	√		√			√	FEM
8	Samui et al.	2011	RVM-based response surface	√		√		√		LEM (Simplified Bishop)
9	Samui et al.	2013	LSSVM-based response surface	√		√		√		LEM (Simplified Bishop)
10	Luo et al.	2012a	Kriging-based response surface	√		√		√	√	FDM
11	Luo et al.	2012b	Kriging-based response surface	√		√		√	√	FEM
12	Ji et al.	2012	Quadratic polynomial without cross terms	√			√	√		LEM (Spencer)
13	Ji and Low	2012	Quadratic polynomial without cross terms		√	√			√	LEM (Ordinary, Spencer)
14	Zhang et al.	2011a	Kriging-based response surface	√		√			√	FDM
15	Zhang et al.	2011b	Quadratic polynomial without cross terms		√	√		√	√	LEM (Morgenstern–Price)
16	Zhang et al.	2013a	Classical RSM without cross terms, quadratic polynomial without cross terms, Kriging-based response surface	√		√			√	LEM (Simplified Bishop)
17	Zhang et al.	2013b	Quadratic polynomial without cross terms		√	√			√	LEM (Simplified Bishop)
18	Li et al.	2013	Updated SVM-based response surface	√		√			√	LEM (Simplified Bishop, Spencer)
19	Tan et al.	2013	Quadratic polynomial	√		√			√	LEM (Morgenstern–Price)
20	Piliounis and Lagaros	2014	NN-based response surface	√		√		√		LEM (Simplified Bishop)
21	Jiang et al.	2014	Hermite polynomial chaos expansion	√			√	√		LEM (Morgenstern–Price)
22	Jiang et al.	2015	Hermite polynomial chaos expansion		√		√	√		LEM (Simplified Bishop)
23	Li et al.	2015a	Quadratic polynomial without cross terms		√		√	√	√	LEM (Simplified Bishop)
24	Li and Chu	2015	Quadratic polynomial without cross terms		√		√		√	LEM (Ordinary)
25	Yi et al.	2015	PSO Kriging based response surface, classical RSM without cross terms	√		√			√	FDM
26	Kang et al.	2015	GPR-based response surface	√		√			√	LEM (Simplified Bishop)
27	Kang and Li	2015	ABC-SVR response surface	√		√			√	LEM (Simplified Bishop)

Note: SVM = support vector machine; ANN = artificial neural network; RBFN = radial basis function neural network; RVM = relevance vector machine; LSSVM = least square support vector machine; NN = neural networks; PSO = particle swarm optimization; GPR = Gaussian process regression; LEM = limit equilibrium method; FEM = finite element method; FDM = finite difference method. ABC-SVR = artificial bee colony algorithm optimized support vector regression.