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System reliability analysis of layered soil slopes using fully specified slip surfaces and genetic algorithms

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ABSTRACT

This paper presents a new approach to identify the fully specified representative slip surfaces (RSSs) of layered soil slopes and to compute their system probability of failure, $P_{f,s}$. Spencer's method is used to compute the factors of safety of trial slip surfaces, and the first order reliability method (FORM) is employed to efficiently evaluate their reliability. A custom-designed genetic algorithm (GA) is developed to search all the RSSs in only one GA optimization. Taking advantage of the system aspects of the problem, such RSSs are then employed to estimate the reliability of the slope system, and a proposed linearization approach—based on first or second order (FORM or SORM) reliability information about the identified RSSs—is used to efficiently estimate $P_{f,s}$. Three typical benchmark-slopes with layered soils are adopted to demonstrate the efficiency, accuracy and robustness of the suggested procedure, and advantages of the proposed method with respect to alternative methods are discussed. Results show that the proposed approach provides reliability estimates that improve previously published results, emphasizing the importance of finding good RSSs—and, especially, good (probabilistic) critical slip surfaces that might be circular or non-circular—to obtain good estimations of the reliability of soil slope systems.

1. Introduction

Probabilistic analyses of slope stability have become a topic of increasing interest in the geotechnical literature (see e.g., Oka and Wu, 1990; Chowdhury and Xu, 1993, 1995; Griffiths and Fenton, 2004; Hong and Roh, 2008; Ching et al., 2009; Cho, 2009; Ching et al., 2010; Huang et al., 2010; Low et al., 2011; Wang et al., 2011; Zhang et al., 2011; Ji and Low, 2012; Cho, 2013; Li et al., 2013; Zhang et al., 2013; Li and Chu, 2014; Li et al., 2014). Some studies have focused on the search for "critical" slip surfaces; i.e., slip surfaces with a minimum reliability index, β. But, as pointed out by Cornell (1971), slope stability is clearly a system problem: a slope may fail along many slip surfaces, and the probability of failure of the slope system, $P_{f,s}$, is larger than the probability of failure of any individual slip surface.

Simulation methods, such as Monte Carlo simulation (MCS) can naturally deal with the system aspects of slope stability to compute $P_{f,s}$. But they could be computationally expensive, particularly for slopes with small probabilities of failure (Ching et al., 2009). More efficient simulation methods, such as "subset simulation" (Wang et al., 2011) and Importance Sampling (IS) (Ching et al., 2009, 2010), have also been employed to solve the system's reliability problem. However, to obtain unbiased estimators of $P_{f,s}$, several previous approaches based on these simulation methods (see e.g., Ching et al., 2010; Wang et al., 2011) need to search the critical slip surface in many of the simulations conducted, hence significantly increasing their computational cost; whereas other approaches, to avoid the optimization problem of finding critical slip surfaces, are based on randomly generated trial slip surfaces (see e.g., Ching et al., 2009), which may therefore miss the critical one, hence providing biased estimators of $P_{f,s}$.

An alternative approach to overcome these drawbacks is to take advantage of the special nature of soil slopes. Zhang et al. (2011) pointed out that the factors of safety of potential slip surfaces are 'correlated'. $P_{f,s}$ can be therefore estimated considering only a few weakly correlated representative slip surfaces (RSSs); in particular, only those RSSs with a higher contribution to $P_{f,s}$ are needed.

The identification of the RSSs of a layered slope system is a challenging task, and several approaches have been therefore proposed to identify them. Some are based on randomly generating many potential slip surfaces (see e.g., Zhang et al., 2011; Li and Chu, 2014; Li et al., 2014), being again likely to miss the critical one; whereas others employ formal optimization methods—such as the extended Hassan and Wolff method (Ji and Low, 2012; Zhang et al., 2013) and the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm with barrier optimal method (Cho, 2013)—but mostly considering circular slip surfaces only. This means that they will often miss the critical slip surface, particularly in the presence of weak seams (see Zolfaghari et al., 2005); and missing the critical slip surface is likely to make them underestimate $P_{f,s}$. Some researchers have considered non-circular slip surfaces: Ching et al.

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(2010) employed the procedure suggested in Greco (1996) to identify non-circular critical slip surfaces in weak seams; and Wang et al. (2011) mentioned that their method can be extended to consider non-circular slip surfaces. But as discussed above, these previous approaches could be computationally expensive. Similarly, Cho (2013) briefly described the use of non-circular slip surfaces in a horizontally-layered slope using the method in Kim and Lee (1997), but with an optimization algorithm that as we show later, failed to identify the critical slip surfaces. For those reasons, developing improved algorithms to identify the RSSs and to compute $P_{f,s}$ is still needed.

This paper presents a new approach to estimate the system's probability of failure of slopes in soils formed by layers with uncertain properties. (Spatial variability within layers is not considered.) The approach makes use of "fully specified slip surfaces". (The term "fully specified slip surface" is employed herein to denote slip surfaces—that could be circular or non-circular but with concave shape—whose geometry is described by the coordinates of all their individual nodes.) Our work extends the genetic algorithm (GA) designed by Jurado-Piña and Jimenez (2014) for the search of fully specified slip surfaces with minimum factor of safety, to consider that the fitness of trial slip surfaces is given by their reliability computed with the First Order Reliability Method (FORM); and it integrates such extended GA with a modified version of the algorithm proposed by Zhang et al. (2011), to select, during the search process, the RSSs that contribute most to the system reliability. A linearization approach is proposed so that the identified RSSs, together with their FORM reliability information, can be used to efficiently estimate the system probability of failure, $P_{f,s}$. Finally, the performance of the proposed method is tested using three different example cases based on typical benchmark-slopes from the literature, obtaining $P_{f,s}$ estimates that improve previously published results.

2. Reliability of slope systems

2.1. Introduction

The reliability of a slope in a layered soil can be modeled as a series system: i.e., as a system that fails when any of its 'components' (or possible slip surfaces) fails. An infinite number of slip surfaces is possible in theory, but only a few are needed to explain the overall $P_{f,s}$ — they are the representative slip surfaces, or RSSs. In our approach, the reliability analysis of a slope system requires three 'tools': (*i*) a first one to compute the performance of individual slip surfaces (i.e., their 'failure' or 'non-failure' states as given by their limit state functions, LSFs), and their reliability (i.e., their probability of failure) given an uncertain characterization of properties; (*ii*) a second one to identify the RSSs; and (*iii*) a third one to compute the reliability of the (series) system.

Methods employed in this paper as tools (*i*) and (*iii*)—i.e., to compute the reliability of an individual slip surface and of a series system—are discussed below. Section 3 describes the developed methodology employed as tool (*ii*); i.e., to identify the RSSs using a new GA approach.

2.2. Computing the reliability of an individual slip surface

Spencer's method, as implemented in SLOPE8R (Duncan and Song, 1984) with minor modifications to improve convergence, is used in this research to compute the factors of safety of slopes with fully specified slip surfaces. Spencer's method considers parallel inter-slice forces, satisfies both moment and force equilibrium, and is applicable to failure surfaces of any shape (circular and non-circular), hence being the "simplest of the complete equilibrium procedures for calculating the factor of safety" (Duncan and Wright, 2005).

For a slope with a set of given uncertain (random) input variables, $\mathbf{x}$, characterized by their joint probability density function (PDF), $f_x(\mathbf{x})$, reliability methods aim to compute the probability of failure of the slope, as given by the following integral:

$$P_f = P\{G(\mathbf{x}) \le 0\} = \int_{G(\mathbf{x}) \le 0} f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} \tag{1}$$

where $G(\mathbf{x})$ is the limit state function of the slope, LSF, defined as $G(\mathbf{x}) = F_S(\mathbf{x}) - 1$, with F_S being the factor of safety of the slope computed with Spencer's method (note that $G(\mathbf{x}) \le 0$ indicates "failure").

Directly computing the integral in Eq. (1) is challenging, particularly when the LSFs are implicit. The first order reliability method (FORM) was developed to more easily solve Eq. (1). To that end, the original LSF in physical space can be transformed to the standard normal space (see Der Kiureghian (2005) for details), hence providing a transformed LSF, $g(\mathbf{u}) = 0$. Then, FORM uses a linear approximation to $g(\cdot) = 0$ at the "design point", $\mathbf{u}^*$, which can be obtained solving the following constrained optimization problem:

$$\begin{cases} \min\limits_{\mathbf{u}} \|\mathbf{u}\| \\ \text{subject to} \quad g(\mathbf{u}) = 0 \end{cases} \tag{2}$$

where $\|\cdot\|$ is the norm of a vector. Once $\mathbf{u}^*$ is obtained, its (minimum) distance to the origin is called the reliability index, $\beta = \|\mathbf{u}^*\|$, and the probability of failure can be computed as $P_f \approx \Phi(-\beta)$, where $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard normal distribution.

Several algorithms have been proposed to find the design point that solves the FORM problem in Eq. (2) (see e.g., Hasofer and Lind, 1974; Rackwitz and Flessler, 1978; Santos et al., 2012). In this study, the improved Hasofer–Lind Rackwitz–Flessler (iHLRF) algorithm (Zhang and Der Kiureghian, 1995)—a tradeoff between efficiency and accuracy—is employed.

FORM also provides 'unit direction vectors' needed to compute correlations between the different LSFs involved in a system. In particular, the correlation coefficient between two LSFs (or slip surfaces), $\rho_{i,j}$, can be computed as $\boldsymbol{\alpha}_i^{\mathrm{T}}\boldsymbol{\alpha}_j$, where $\boldsymbol{\alpha}$ is the unit direction vector at the design point, given by $\boldsymbol{\alpha} = \mathbf{u}^*/\|\mathbf{u}^*\|$.

2.3. A linearization approach to compute the reliability of a series system

Whenever a set of m RSSs of a layered slope system are identified during the search process using the procedure proposed herein (see Section 3), the probability of failure of the series system can be estimated, efficiently and accurately, using the FORM-results (β and $\boldsymbol{\alpha}$) for the LSFs of the identified RSSs, as (Hohenbichler and Rackwitz, 1982; Zeng and Jimenez, 2014):

$$P_{f,s} = 1 - P^{1,\ldots,m} \approx 1 - \Phi_m(\boldsymbol{\beta}, \mathbf{R}) \tag{3}$$

where $P^{1,\ldots,m}$ is the probability of the intersection of safe domains for the m RSSs (and their associated LSFs) considered; $\Phi_m(\boldsymbol{\beta}, \mathbf{R})$ is the CDF of the m-dimensional standard normal distribution, evaluated at the vector of reliability indices, $\boldsymbol{\beta} = [\beta_1, \beta_2, \cdots, \beta_m]$, and with correlation matrix, $\mathbf{R}$, given by $\mathbf{R}[i,j] = \rho_{i,j}$, $i = 1, \ldots, m$ and $j = 1, \ldots, m$. (Φ_m is readily available in standard mathematical packages; for instance, it can be computed efficiently by MATLAB's pre-complied function **mvncdf**.) In general, the reliability index corresponding to the ith LSF, β_i, can be computed using FORM. Although, as we will see, reliability indices computed with the second order reliability method (SORM) (Phoon, 2008) improve the computed estimates of $P_{f,s}$ and might be needed when the LSFs are highly non-linear (Zeng and Jimenez, 2014) (note, however, that many slope stability problems are expected to be quite linear (see e.g., Zhang et al., 2011; Li and Chu, 2014; Zeng and Jimenez, 2014); for instance, the LSF associated to Bishop's simplified method for cohesive soils under short-term conditions and with normally distributed and uncorrelated cohesions is *exactly* linear).