Engineering Geology 163 (2013) 144–152

Contents lists available at ScienceDirect

Engineering Geology

journal homepage: www.elsevier.com/locate/enggeo

Quantification of uncertainties in soil–water characteristic curve associated with fitting parameters

Qian Zhai [a], Harianto Rahardjo [b,*]

[a] Division of Infrastructure Systems and Maritime Studies, School of Civil and Environmental Engineering, Nanyang Technological University, Block N1, B4B-07, Nanyang Avenue, Singapore 639798, Singapore
[b] Division of Infrastructure Systems and Maritime Studies, School of Civil and Environmental Engineering, Nanyang Technological University, Block N1, #1B-36, 50 Nanyang Avenue, Singapore 639798, Singapore

ARTICLE INFO

Article history:
Received 30 November 2012
Received in revised form 5 April 2013
Accepted 26 May 2013
Available online 6 June 2013

Keywords:
Soil–water characteristic curve
Variance
Uncertainty
Confidence limits

ABSTRACT

Soil–water characteristic curve (SWCC) is commonly expressed using best fit equations with several fitting parameters. These fitting parameters are determined by best fitting experimental data with the best fit equations. Residual errors always exist after the regression procedure for the determination of these fitting parameters. Statistical theory suggests that uncertainties of the determined SWCC can be estimated from the variance of these fitting parameters and the residual errors. In this paper, equations for the confidence limits of the best fitted SWCC are developed to quantify the uncertainties in the determined SWCC associated with the fitting parameters. Applications of the confidence limits in evaluating the performance of best fit equations and suggestion for experimental measurements are presented in this paper.

1. Introduction

Soil–water characteristic curve (SWCC) is a graphical relationship that shows the relationship between the amount of water in a soil (i.e. gravimetric water content w, volumetric water content θ_w or degree of saturation S (Fredlund and Rahardjo, 1993)) and matric suction ψ. As introduced by Fredlund (2006), the entire suction range of the SWCC can be divided into three zones such as boundary effect zone, transition zone and residual zone and they are separated by air-entry value and residual suction as illustrated in Fig. 1.

SWCC is commonly expressed using best fit equations with several fitting parameters. The fitting parameters are determined from limited experimental data by applying a curve fitting technique by minimizing the sum of squared-errors (i.e. $\Sigma w_i * (\theta_i - \theta'_i)^2$, where: θ_i is the measured volumetric water content, θ'_i is the modeled volumetric water content, and w_i is the weighting factor as suggested by Leong and Rahardjo, 1997). Equations for correlation of these fitting parameters and SWCC variables (i.e. air-entry value, slope at the inflection point, residual suction and residual volumetric water content) were developed by Zhai and Rahardjo (2012) as an alternative to the traditional graphical method. In this paper, equations to quantify the uncertainties in SWCC associated with these fitting parameters are developed.

Residual error (i.e. Sum of squared errors) always exists after the regression procedure. Statistical theory (Benjamin and Cornell, 1970) suggests that the uncertainties of SWCC can be estimated from the coefficient of correlation equation and residual error. In this paper, equations for the determination of the variance of these fitting parameters and subsequently confidence limits of the best fitted SWCC and SWCC variables are derived for Fredlund and Xing's (1994) equation.

2. Literature review

Different best fit equations, such as proposed by Brooks and Corey (1964), Van Genuchten (1980), Fredlund and Xing (1994), Kosugi (1996) and Pedroso et al. (2009), have been developed to describe SWCC that relates the amount of water in a soil to the matric suction. Leong and Rahardjo (1997) concluded that Fredlund and Xing's (1994) equation was the best fit equation which could be used for a wide range of soil over the entire range of matric suction. Therefore, in this paper Fredlund and Xing's (1994) equation is selected for best fitting the experimental data for the determination of the SWCC:

$$\theta = C(\psi)\frac{\theta_s}{\left\{\ln\left[e+\left(\frac{\psi}{a}\right)^n\right]\right\}^m} = \left[1-\frac{\ln\left(1+\frac{\psi}{C_r}\right)}{\ln\left(1+\frac{10^6}{C_r}\right)}\right]\frac{\theta_s}{\left\{\ln\left[e+\left(\frac{\psi}{a}\right)^n\right]\right\}^m} \quad (1)$$

where,

a, n, m — fitting parameters;
C_r — an input value related to the residual suction which can be rough estimated as $C_r = 1500$ kPa for most cases.
θ_s — saturated volumetric water content.

* Corresponding author. Tel.: +65 6790 5246; fax: +65 6791 0676.
E-mail addresses: qzhai1@e.ntu.edu.sg (Q. Zhai), chrahardjo@ntu.edu.sg (H. Rahardjo).
URL: http://research.ntu.edu.sg/expertise/academicprofile/pages/StaffProfile.aspx?ST_EMAILID=CHRAHARDJO (H. Rahardjo).

0013-7952/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.enggeo.2013.05.014

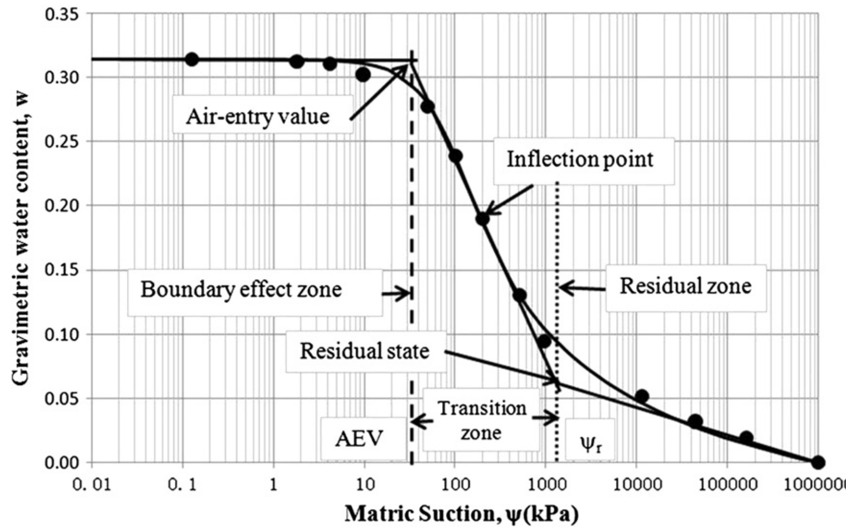

Fig. 1. Illustration of different zones of de-saturation as defined by the soil–water characteristic curve (after Fredlund, 2006).

There are only three unknown fitting parameters (i.e. a, n and m) in Fredlund and Xing's (1994) equation, C_r is an input value and not a fitting parameter. Zhai and Rahardjo (2012) defined Fredlund and Xing's (1994) equation with correction factor C(ψ) as Method A and Fredlund and Xing's (1994) equation with correction factor C(ψ) = 1, which was suggested by Leong and Rahardjo (1997), as Method B.

Mishra et al. (1989) and Phoon et al. (2010) indicated that a first-order error analysis was a reasonable approximation for estimation of uncertainty in a predictive model in view of the lack of complete measurements for calibration data set which would enable more direct assessment. The first-order error analysis was based on Taylor expansion around the mean values of parameters by assuming small parameter perturbations and negligible higher-order terms. On the other hand, laboratory measurement of SWCC is very time consuming and costly because the equilibrium time for each data point can be very long especially for fine-grained soils. Therefore, it is very difficult to obtain sufficient experimental data for direct assessment of uncertainty in the determined SWCC while the first-order error analysis provides an indirect assessment of uncertainty. In this paper the first-order error analysis is adopted to evaluate the uncertainty in SWCC associated with the fitting parameters which are determined from limited experimental data.

Beck and Arnold (1977), Kool et al. (1987), Mishra et al. (1989), and Mishra and Parker (1989) indicated that the covariance matrix C could be used to represent the variances of estimated parameters and also introduced the procedure for estimation of the error covariance matrix C using the first-order error analysis approach as illustrated below.

$$C = E\left[\left(\tilde{b}-b\right)\left(\tilde{b}-b\right)^T\right] \approx \frac{s^2\left(J^TJ\right)^{-1}}{M-P} \quad (2)$$

where:

- $\tilde{b}$ is the vector of estimated parameters
- b is the vector of true parameters
- E denotes statistical expectation
- s^2 is the sum of squared-error;
- M is the number of experimental data points;

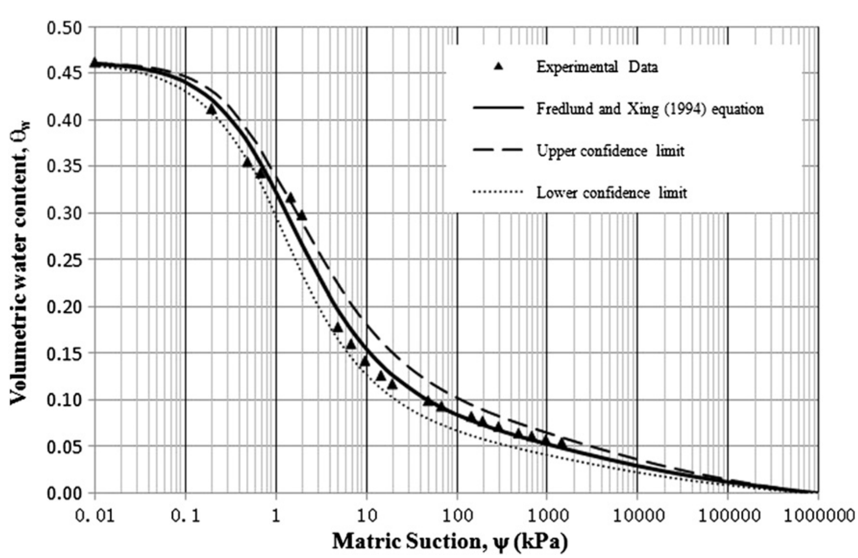

Fig. 2. Illustration of confidence limits of the best fitted SWCC.