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Research Paper

Thermal radiation analysis for small satellites with single-node model using techniques of equivalent linearization

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HIGHLIGHTS

- Linearization criteria are presented for a single-node model of satellite thermal.
- A nonlinear algebraic system for linearization coefficients is obtained.
- The temperature evolutions obtained from different methods are explored.
- The temperature mean and amplitudes versus the heat capacity are discussed.
- The dual criterion approach yields smaller errors than other approximate methods.

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ABSTRACT

In this paper, the method of equivalent linearization is extended to the thermal analysis of satellite using both conventional and dual criteria of linearization. These criteria are applied to a differential nonlinear equation of single-node model of the heat transfer of a small satellite in the Low Earth Orbit. A system of nonlinear algebraic equations for linearization coefficients is obtained in the closed form and then solved by the iteration method. The temperature evolution, average values and amplitudes versus the heat capacity obtained by various approaches including Runge–Kutta algorithm, conventional and dual criteria of equivalent linearization, and Grande's approach are compared together. Numerical results reveal that temperature responses obtained from the method of linearization and Grande's approach are quite close to those obtained from the Runge–Kutta method. The dual criterion yields smaller errors than those of the remaining methods when the nonlinearity of the system increases, namely, when the heat capacity varies in the range $[1.0, 3.0] \times 10^4$ J K^{-1}.

1. Introduction

Thermal analysis is one of the most important problems in both processes of designing and manufacturing a satellite. It ensures that the satellite will operate in appropriate ranges of temperature in its orbit [1,2]. New techniques developed for satellite thermal problems are integrated in commercial software [1,3] with complex numerical algorithms. On the aspect of analytical calculations, the analysis of satellite thermal problems is difficult because of the presence of complicated nonlinear terms in thermal models. Over decades, several techniques in analyzing satellite thermal have been proposed, for instance, the Fourier analysis [4] and techniques of linearization method [5–8]. In Ref. 4, Oshima and Oshima carried out some analytical calculations for thermal design of spacecraft in several special cases related to real spacecraft structures. Grande et al. [6] studied a technique of linearization for a problem of thermal models of satellites in the Low Earth Orbit (LEO). Within their technique, the direct integration of heat balance equations of a two-node reduced model was used to obtain linearized second-order ordinary differential equations similar to the classical case of the forced vibration of damped systems. In Ref. 7, Gaite et al. analyzed a simple model of the heat transfer from a small satellite orbiting around a solar system planet. By using the perturbation and numerical methods, the authors showed that the temperature response of the satellite model approaches an attracting limit cycle. Gaite [8] investigated a system of differential equations of lumped-parameter models of spacecraft thermal control. His study arrived at the result that the spacecraft temperature approaches to a steady-state if the heat input is constant, and approaches to a limit cycle if the heat input is periodic. The result was then generalized to the many-node system of spacecraft thermal problem. Recently, Gaite and Fernández-Rico [9] have developed a heuristic linearization

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method to get a system of linearized differential equations for the nodal temperatures of a lumped-parameter thermal model of a spacecraft moving in a closed orbit. The ten-node model of a moon-orbiting satellite was then applied in order to elucidate the proposed approach.

This study is devoted to the extension of the method of equivalent linearization (EQL) to the analysis of satellite thermal problem. The EQL method is well-known in the theory of random vibration [10–13]. It was extended from Krylov and Bogoliubov's linearization method [14] for deterministic vibrations. The fundamental idea of the EQL method lies on the replacement of nonlinear terms of the original nonlinear system under external excitations by linear terms of the equivalent linear system under the same excitations, in which the equivalent linearization coefficients are found from a specified criterion, such as mean-square error criterion [11] and moment criterion [15]. On the development of the EQL techniques, readers can be seen in detail in the book of Roberts and Spanos [16], and papers published recently [17–21].

It is the first time that the equivalent linearization method is extended to nonlinear problems of satellite thermal models using the conventional linearization criterion and dual approach of EQL developed recently for single and multi-degree-of-freedom systems [20,21]. The conventional and dual criteria of EQL are developed to a single-node nonlinear thermal model of small satellite. Numerical calculations for the system of satellite thermal are carried out to show the high accuracy of the dual criterion of EQL when the nonlinearity is increasing.

2. Single-node thermal model for small satellites in LEO

The equation of thermal balance for a small satellite with the single-node thermal model can be taken in the form [7]

$$C\dot{T} = -A_{sc}\varepsilon\sigma T^4 + Q_s f_s(\nu t) + Q_a f_a(\nu t) + Q_p, \tag{1}$$

where $T = T(t)$ is the temperature of the satellite and C is the thermal capacity. The notation A_{sc} represents the surface area of the node in the model; ε is the emissivity; $\sigma = 5.67\times10^{-8}$ W m^{-2} K^{-4} is the Stefan–Boltzmann constant. The quantity Q_s is a constant and determined from the following expression

$$Q_s = G_s A_{sp}\alpha_s, \tag{2}$$

where G_s is the mean solar irradiation and A_{sp} is the satellite surface projected in the Sun's direction. In practice, for a small compact satellite, this area is assumed to be constant. The coefficient α_s represents the solar absorptivity of the satellite surface area. The solar heat loading remains constant during the illumination period P_{il}. While the satellite is illuminated, the solar irradiation thermal loading $Q_s f_s(\nu t)$ differs from zero. Against, this loading will vanish as the satellite is in the fraction of orbit in eclipse depending on the minimum angle between the orbital plane and the solar vector [6,7]. One has a mathematical representation for $f_s(\nu t)$ in an orbital period P_{orb} as follows

$$f_s(\nu t) = \begin{cases} 1 & \text{if} \quad 0 \le \nu t \le \mu\pi \quad \text{and} \quad \left(1-\dfrac{1}{2}\mu\right)2\pi \le \nu t \le 2\pi, \\ 0 & \text{if} \quad \mu\pi < \nu t < \left(1-\dfrac{1}{2}\mu\right)2\pi, \end{cases} \tag{3}$$

where $\mu = P_{il}/P_{orb}$ is the ratio of the illumination period to the orbital period, $\nu = 2\pi/P_{orb}$.

The albedo radiation $Q_a f_a(\nu t)$ in Eq. (1) must be taken into account because it affects the thermal behavior of the satellite during the orbit. The quantity Q_a is determined by [6]

$$Q_a = aG_s A_{sc}F_{scp}\alpha_s, \tag{4}$$

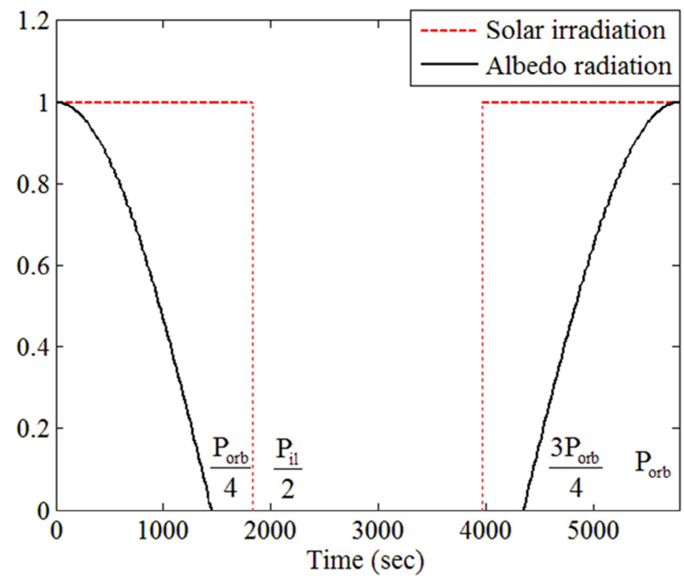

Fig. 1. Variations of the solar and albedo radiation in an orbital period.

in which the albedo factor a is estimated from the experimental data. For the Earth, the albedo factor can be taken from 0.31 to 0.39. Its value is highly dependent on the local surface and atmospheric properties of the Earth [22]. F_{scp} is the view factor from the whole satellite to the Earth. The albedo loading depends on the orbit altitude and the solar zenith angle at the satellite position. One can estimate the albedo loading as a function of time t. The mathematical form of $f_a(\nu t)$ for an orbital period is as follows

$$f_a(\nu t) = \begin{cases} \cos(\nu t) & \text{if} \quad 0 \le \nu t \le \dfrac{\pi}{2} \quad \text{and} \quad \dfrac{3\pi}{2} \le \nu t \le 2\pi, \\ 0 & \text{if} \quad \dfrac{\pi}{2} < \nu t < \dfrac{3\pi}{2}. \end{cases} \tag{5}$$

The graphs of $f_s(\nu t)$ and $f_a(\nu t)$ are plotted in Fig. 1 for an orbital period P_{orb} (see also Ref. 6). The planetary infrared radiation Q_p in Eq. (1) can be evaluated as

$$Q_p = \varepsilon A_{sc}F_{scp}\sigma T_p^4, \tag{6}$$

where T_p is the planetary black-body equivalent temperature. In the next section, we will find approximate solutions of Eq. (1) using the method of equivalent linearization.

3. Techniques of equivalent linearization

The equation of thermal balance (1) is transformed in the following non-dimensional form

$$\frac{d\theta}{d\tau} = -\theta^4 + \gamma_1 f_s(\tau) + \gamma_2 f_a(\tau) + \gamma_3, \tag{7}$$

where $\theta = \theta(\tau)$ is the dimensionless temperature response of the dimensionless time τ, and it is denoted as

$$\theta = T\left(\frac{\tau}{\nu}\right)\Big/\beta, \quad \beta = [C\nu/(A_{sc}\varepsilon\sigma)]^{1/3}, \quad \tau = \nu t, \quad \nu = 2\pi/P_{orb},$$
$$\gamma_1 = Q_s/(\beta C\nu), \quad \gamma_2 = Q_a/(\beta C\nu), \quad \gamma_3 = Q_p/(\beta C\nu) \tag{8}$$

Eq. (7) can be solved numerically to obtain the thermal behavior of the satellite using the Runge–Kutta algorithm. In Ref. 6, Gaite