Applied Thermal Engineering 100 (2016) 392–397

Contents lists available at ScienceDirect

Applied Thermal Engineering

journal homepage: www.elsevier.com/locate/apthermeng

Research Paper

A novel analytical solution for the transmissivity of curved transparent surfaces with application to solar radiation

Wael M. El-Maghlany *

Department of Mechanical Engineering, Faculty of Engineering, Alexandria University, Alexandria, Egypt

HIGHLIGHTS

- A novel method to calculate transmissivity of curved transparent surfaces is developed.
- Transmissivity is calculated as a function of geographical location and surface geometry.
- The novel analytical method derives direct and diffuse radiation coefficients.
- Total transmitted solar energy through a curved surface can be directly calculated.

ARTICLE INFO

Article history:
Received 28 October 2015
Accepted 15 February 2016
Available online 26 February 2016

Keywords:
Transmissivity
Solar energy
Curved surfaces

ABSTRACT

In this article, a novel mathematical expression for calculating the transmissivity of curved transparent surfaces due to sun rays for both direct and diffuse radiation is presented. An analytical solution is performed for the transmissivities of both direct and diffuse radiation – τ_{gn} and τ_{gd} – as functions of the latitude (ϕ), orientation (γ), and the curved surface aspect ratio ($P = b/a$) to derive new parameters that are defined as direct and diffuse radiation coefficients ($\overline{\Gamma_n}$ and $\overline{\Gamma_d}$). Hence, such parameters can be directly used to evaluate the total transmitted solar energy through curved transparent surfaces that are widely used in greenhouses as well as new designs of solar collectors with curved glass covers. The proposed mathematical expression should be of significant value to the design of solar equipment in a wide range of applications.

1. Introduction

Glass and transparent materials are the main components of the solar collectors, greenhouses, and many other industrial applications that utilize solar energy. The conventional shape of the glass is a flat surface, and the transmissivity as a function of incidence and inclination angles was reported in the literature [1–5]. Numerous studies investigated the modeling of solar thermal radiation in enclosures [6–11]. Waide and Norton [12] developed a model to determine the transmittance of glazing for specific direct and diffuse radiation. They obtained the transmittance of glazing in different planes by using meteorological data available for the place. Furbo and Jivan Shah [13] established a correlation for calculating the total transmittance for both normal glass and another glass with anti-reflection coating for the different solar angle. Oliveira et al. [14] introduced models to estimate the diffused radiation fraction at a different time for a given location using metrological and environmental data. Recently, a novel decent design for the solar collector with curved glass cover is represented by Rodríguez-Sánchez et al. [15], and their valuable work was the first trial in the way to the simulation of curved surface analytically; they analyzed a quarter of a circle due to consideration of symmetry in both horizontal and vertical positions, and the results were for the total collected solar energy on the surface of the solar collector. Another novel study was represented by Tanaka et al. [16] for a curved surface with different geometries. However, their study was limited to specified elliptic shapes of the glass cover and the total collected solar energy was only on the surface of the solar collector, not the transmitted energy through the curved elliptic surface. Such recent research directions on utilizing curved glass surfaces in solar energy equipment have revealed that there is a need to have a theoretical framework to analyze solar transmissivity through curved surfaces, which is not present in the literature. In the present work, a new mathematical analysis framework for the transmissivity through the curved surface is developed and implemented, which should be of vital importance for the applications of solar equipment utilizing such curved surfaces. Such applications include roof-top solar collectors, buildings facades, as well as agricultural greenhouses. Hence, the present work compliments the work in [15,16] in order to get the transmitted solar energy through glass

* Tel.: +201004090302; fax: +20452230646.
E-mail address: elmaghlany@alexu.edu.eg; elmaghlany@yahoo.com.

http://dx.doi.org/10.1016/j.applthermaleng.2016.02.062

or transparent materials for different curved surfaces for specific elliptic shape.

2. Modeling of direct and diffuse radiations on curved surface

Fig. 1 illustrates the possible shapes of the elliptic curved surface with the polar coordinate system. In this study, the analysis for the curved surface will be focused on the horizontal position, when the main axis of the curved surface is horizontal and the azimuth of the surface can have any value.

The relation between the radius r of elliptic shape and the angle α is set as [16]

$$r\sqrt{a^2\cos^2\alpha+b^2\sin^2\alpha}=ab \tag{1}$$

$$r=\frac{b}{\sqrt{P^2\cos^2\alpha+\sin^2\alpha}} \tag{2}$$

The slope of the elliptic curved surface at any position α representing the inclination angle β after simplification is given by [16]

$$\beta=\tan^{-1}(P^2\cot\alpha) \tag{3}$$

The direct radiation on the elliptic curved surface is not uniform because the slope of the surface β varies with the angle α in confound manner according to equation (3).

The transmitted direct radiation through the curved surface can be obtained by:

$$G_n=G_{bn}\overline{\Gamma_n} \tag{4}$$

where:

$$\overline{\Gamma_n}=\frac{\int_0^{\frac{\pi}{2}}\tau_{gn}r\cos\theta d\alpha}{\int_0^{\frac{\pi}{2}}rd\alpha} \tag{5}$$

The transmissivity of the glass cover due to direct radiation, τ_{gn} (θ), is the function of the incident angle of direct radiation θ and may be presented as [1].

$$\tau_{gn}=2.642\times\cos\theta-2.163\times\cos^2\theta-0.32\times\cos^3\theta+0.719\times\cos^4\theta \tag{6}$$

The angle θ is the incident angle of direct radiation to tilted surface and related to the declination δ, the latitude ϕ, the slope of the surface β, the azimuth of the surface γ and the hour angle Ω as given below:

$$\cos\theta=M\cos\beta+N\sin\beta \tag{7}$$

where

$$\begin{aligned}M&=\sin\delta\sin\varphi+\cos\delta\cos\varphi\cos\Omega\\ N&=\cos\delta\sin\varphi\cos\gamma\cos\Omega-\sin\delta\cos\varphi\cos\gamma+\cos\delta\sin\gamma\sin\Omega\end{aligned} \tag{8}$$

In the above equations, north is positive for ϕ, due south is zero for γ, with east negative and west positive, and the hour angle Ω is negative in the morning and positive in the afternoon. Substituting equations (2), (3), and (7) with equation (5), it will be

$$\overline{\Gamma_n}=\left[\frac{\int_0^{\frac{\pi}{2}}\frac{(2.642\times\cos^2\theta-2.163\times\cos^3\theta-0.32\times\cos^4\theta+0.719\times\cos^5\theta)}{\sqrt{P^2\cos^2\alpha+\sin^2\alpha}}d\alpha}{\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{P^2\cos^2\alpha+\sin^2\alpha}}d\alpha}\right] \tag{9}$$

Isotropic sky models will be used for estimation of diffuse radiation. Isotropic sky models are simple models that assume a uniform distribution of the diffuse radiation over the sky dome and circumsolar and horizontal brightening parts are considered to be zero. According to the simple model of an isotropic sky, the diffuse radiation can be obtained according to:

$$G_d=G_{dn}\overline{\Gamma_d} \tag{10}$$

where:

$$\overline{\Gamma_d}=\frac{\int_0^{\frac{\pi}{2}}\tau_{gd}r(1+\cos\beta)d\alpha}{\int_0^{\frac{\pi}{2}}rd\alpha} \tag{11}$$

The transmissivity of the glass cover due to diffuse radiation, τ_{gd} (β), is the function of the glass cover angle, β, and may be presented as [1].

$$\begin{aligned}&\tau_{gd}(\beta)=0.667-2.05\times10^{-3}\beta-2.03\times10^{-5}\beta^2,\quad with\quad \beta(\text{deg})\\ &\tau_{gd}(\alpha)=0.667-2.05\times10^{-3}\tan^{-1}(P^2\cot\alpha)\\ &\qquad-2.03\times10^{-5}\left(\tan^{-1}(P^2\cot\alpha)\right)^2\end{aligned} \tag{12}$$

Substituting equations (2) and (3) with equation (11), it will be

$$\overline{\Gamma_d}=\left[\frac{\int_0^{\frac{\pi}{2}}\tau_{gd}(\alpha)\frac{\left(1+\cos\left(\tan^{-1}(P^2\cot\alpha)\right)\right)}{\sqrt{P^2\cos^2\alpha+\sin^2\alpha}}d\alpha}{\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{P^2\cos^2\alpha+\sin^2\alpha}}d\alpha}\right] \tag{13}$$

The integrals in equations (9) and (13) could be solved for different curved surface aspect ratios P in the range of $0\le P\le\infty$; it will be noted that $P=0$ represents the flat horizontal surface, $P=\infty$ represents the flat vertical surface, and $P=1$ represents a quarter of a circle. On behalf of practical applications, the aspect ratios will be in the range $0\le P\le 10$. The results of the complicated solution to get the values of both $\overline{\Gamma_n}$ and $\overline{\Gamma_d}$, to be directly used in the valuation of the total transmitted solar energy through the curved surface, are displayed for different M and N (functions of the latitude ϕ, the orientation γ, and the hour Ω), as well as the different geometries (P) of the curved surface as follows:

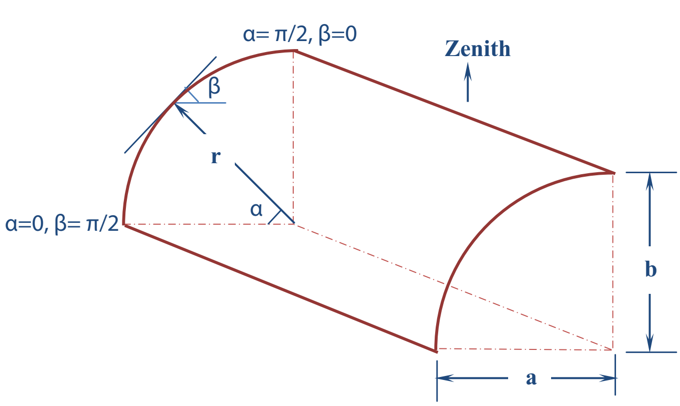

Fig. 1. Polar coordinates for the curved surface.