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Research paper

Preliminary design criteria of Stirling engines taking into account real gas effects

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HIGHLIGHTS

- The theoretical limit of the gas circuit performance considers real gas effects.
- The influences of physical gas properties are analysed.
- Simulations predict the feasibility of operating under real gas effects.
- Equations and correlations are proposed as preliminary design criteria.
- The procedure is supported by an experimental database.

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ABSTRACT

This article deals with the main geometric parameters, operating variables and experimental results of engines with different size and characteristics, as well as simulation predictions at operating conditions for which the working fluid may evidence real gas effects. The concept of dimensionless quasi-static indicated work, introduced previously, is computed assuming both ideal and real gas models. Both values are compared with the Schmidt's model prediction, to evaluate separately how the mechanism simplification and the equation of state affect. The influence of physical gas properties are also analysed regarding the gas circuit performance and mechanical efficiency. Pending of experimental corroboration, some simulations show the possibility of obtaining interesting operating conditions under real gas effects.

Semi-empirical equations and experimental correlations are proposed as preliminary design criteria. The ratio between the dimensionless values of the maximum indicated power (a sort of 'indicated' Beale number) and the quasi-static indicated work is analysed. The procedure to estimate the maximum indicated power is completed by analysing the characteristic Mach number (dimensionless rotation frequency). Analogous procedure is applied for the maximum brake power and corresponding engine speed. Use of dimensionless variables facilitates the generalization of analyses by means of dynamic similarity criteria.

1. Introduction

Potential benefits justify that the Stirling engine has competed with other technologies over the past three decades for applications such as cogeneration at industrial and domestic scales, underwater systems and air independent propulsion of submarines, solar thermal energy conversion, and space vehicles. The commercial battle is full of ups and downs, since the cost of the product is mainly determined by the volume of series production of its components, and by fluctuations in the price of fossil fuels. So far, the Stirling cycle has achieved greater commercial success in their applications as reverse cycle machine, an option that other thermal engines do not have.

The gas circuit and the drive mechanism are the main subsystems of a Stirling engine. The numerical computing power currently available allows the development of models to study

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Nomenclature

A_{cx}	cross-sectional area of space x (m^2)
C_f	local, instantaneous friction factor $= \Delta p \cdot 2\rho(A_{xx}/\dot{m})^2(r_{hx}/L_x)$
c_p	constant pressure heat capacity (J/kg K)
c_v	constant volume heat capacity (J/kg K)
h	convective heat transfer coefficient (W/m^2 K)
k	thermal conductivity (W/m K)
L_x	length of space x (m)
$\dot{m}$	mass flow rate (kg/s)
m_1	reference mass of drive mechanism moving parts (kg)
N_B	Beale number
N_m	characteristic drive mechanism number $= m_1 R T_C/(p_m V_{sw})$
N_{MA}	characteristic Mach number $= n_s V_{Sw}^{1/3}/\sqrt{RT_C}$
N_{ma}	local, instantaneous Mach number $= \dot{m}\sqrt{RT_C}/(pA_{xx})$
$N_{MA,\max}$	Mach number corresponding to the maximum indicated power $= n_{s,\max} V_{Sw}^{1/3}/\sqrt{RT_C}$
$N^*_{MA,\max}$	Mach number corresponding to the maximum brake power $= n^*_{s,\max} V_{Sw}^{1/3}/\sqrt{RT_C}$
$N_{ma,\max}$	local Mach number corresponding to the maximum mass flow rate
N_p	characteristic pressure number $= p_m V_{Sw}^{1/3}/(\mu\sqrt{RT_C})$
N_{PR}	characteristic Prandtl number $= \mu c_p/k$
N_{pr}	local, instantaneous Prandtl number
N_{re}	local, instantaneous Reynolds number $= 4\dot{m} r_{hx}/(\mu A_{xx})$
$N_{re,\max}$	local Reynolds number corresponding to the maximum mass flow rate
N_{SG}	characteristic Stirling number $= p_m/(\mu n_s)$
n_s	engine speed (rev/s)
$n_{s,\max}$	engine speed at maximum indicated power (rev/s)
$n^*_{s,\max}$	engine speed at maximum brake power (rev/s)
N_{st}	local, instantaneous Stanton number $= hA_{xx}/(\dot{m}c_p)$
N_{TCR}	regenerator thermal capacity ratio $= \rho_R c_R/(\rho c_p) = \rho_R c_R T_C/p_m \cdot (\gamma - 1)/\gamma$
N_W	West number
N_α	characteristic regenerator thermal diffusivity number $= \alpha_R/(V_{sw}^{1/3}\sqrt{RT_C})$
p	pressure (Pa)
P_B	brake power (W)
p_{cr}	pressure at the critical point (Pa)
P_{ind}	indicated power (W)
p_m	mean pressure (Pa)
P_{mec}	mechanical power losses $= P_{ind} - P_B$ (W)
p_r	reduced pressure $= p/p_{cr}$
p_{rm}	reduced mean pressure $= p_m/p_{cr}$
R	specific gas constant (J/kg K)
r_{hx}	hydraulic radius of space x (m)
T_{cr}	temperature at the critical point (K)
T_{rx}	reduced temperature at space $x = T_x/T_{cr}$
T_x	working gas temperature at space x (K)
V	volume (m^3)
V_C	swept volume of the compression piston (m^3)
V_{dx}	dead volume of space x (m^3)
V_E	swept volume of the expansion piston (m^3)
V_{sw}	net swept volume $= V_{\max} - V_{\min}$ (m^3)
W_0	quasi-static indicated work per cycle (J)
$\alpha, \beta, \chi, \delta$	dimensionless coefficients of mechanical power losses
α	phase angle (rad)
α_R	regenerator material thermal diffusivity (m^2/s)
α_{cx}	dimensionless cross-sectional area of space $x = A_{xx}/V_{sw}^{2/3}$
$\delta_1 \ldots \delta_n$	dimensionless geometrical parameters, including those characteristic of the drive mechanism
η_{mec}	mechanical efficiency $= P_B/P_{ind}$
$\eta_{mec,\max}$	mechanical efficiency at $n^*_{s,\max}$
Φ	dimensionless factor of linear losses of indicated power
γ	adiabatic coefficient $= c_p/c_v$
κ	swept volume ratio $= V_C/V_E$
κ_s	swept volume ratio according to Iwamoto et al. $= V_{sw}/V_E$
λ_{hx}	dimensionless hydraulic radius of space $x = r_{hx}/V_{sw}^{1/3}$
μ	working fluid viscosity (Pa s)
μ_{dx}	dead volume ratio of space $x = V_{dx}/V_{sw}$
μ_L	lubricant viscosity (Pa s)
$\P_V$	Regenerator volumetric porosity
θ	crank angle (rad)
Ψ	dimensionless factor of quadratic losses of indicated power
ρ	density (kg/m^3)
$\rho_R c_R$	regenerator material volumetric specific heat capacity (J/m^3 K)
τ	temperature ratio $= T_C/T_E$
ζ_B	dimensionless brake power $= P_B/(p_m\ V_{sw}\ n_s)$
ζ_{ind}	dimensionless indicated power $= P_{ind}/(p_m\ V_{sw}\ n_s)$
ζ_{mec}	dimensionless mechanical power losses $= P_{mec}/(p_m\ V_{sw}\ n_s)$
ζ_0	dimensionless quasi-static work per cycle $= W_0/(p_m\ V_{sw})$

Subscripts

C	cooler or compression space
cc	compression cylinder
E	heater or expansion space
ec	expansion cylinder
R	regenerator
x	generic space

Superscripts

R	considering real gas effects
S	according to Schmidt cycle

these subsystems with fewer simplifying assumptions than was possible decades ago. Thus, the development of Stirling engine models capable of being integrated into schemes for the overall techno-economic analysis of applications is still a topic of current interest [1–6]. Because all models must be validated experimentally, the benefit of the advanced models is not greater accuracy but the ability to analyse physical phenomena dependent on variables whose experimental measurement is practically impossible. For now, simple models are needed at the preliminary design stage, while the advanced models can be appropriate for optimisation tasks.

Beale and West numbers were probably the simplest and most commonly used criteria for sizing of engines at the preliminary design stage. However, without prejudice to its historical importance, it should be recognized that both concepts are experimental correlations where great simplifications have been made. So far it