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Research paper

Analysis of the unsteady behavior of an electrical heater using the lock-in technique: Comparison of theory and experiments

Sylvain Lalot*, Bernard Desmet

TEMPO-DF2T, Université de Valenciennes et du Hainaut-Cambrésis, Campus Mont Houy, F-59313 Valenciennes Cedex 9, France

HIGHLIGHTS

- Definition of a characteristic dynamic parameter of an electrical heater.
- Experiments confirm the evolution of this value versus the angular frequency.
- Experimental results are obtained with a disturbance amplitude lower than 0.5 °C.

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ABSTRACT

In the recent years, the lock-in technique has been tested in thermal engineering. Due to its high sensitivity for signal analysis, it has great potentialities for drift detection. Here the aim is more particularly fouling detection. The key point for this technique is the choice of the angular frequency of the exciting signal. It is a tradeoff between getting a high value of the modulus of a complex number, getting a stable value of this modulus in a time as short as possible, and leading to low variations of the temperature of the product passing through the thermal device. The aim of the present study is to show that experimental results obtained using a specialized measurement apparatus are in a good agreement with the theoretical developments for an electrical heater. The first part of the paper is dedicated to a brief presentation of the theory of the lock-in technique for lumped systems. The second part presents the developments for an electrical heater. The third part is dedicated to the presentation of the test rig. Then experimental results are given in the way they have been presented for the theoretical electrical heater. Even if some simplifications have been necessary for the theoretical developments, these experimental results show that the evolution of the modulus is similar to what is expected according to the angular frequency and to the mass flow. A final comparison between experimental results and theoretical computations shows a reasonable agreement for a particular set of parameters.

1. Introduction

The lock-in technique, also called synchronous detection, is widely used in signal processing [1–5] such as analysis of radar signals and thermography. Lock-in detection is used by Xing et al. [6] to obtain a good signal-to-noise ratio for accurate measurement of fine fibers thermal properties by the 3-ω technique. Its main feature is to be able to retrieve a signal when the amplitude of the signal is comparable to and even lower than the amplitude of the noise. In the very recent years, the lock-in technique has been tested in thermal engineering on lumped systems, electrical heaters and heat exchangers [7,8]. Due to its high sensitivity for signal analysis, it has great potentialities for drift detection. Here the aim is more particularly fouling detection. The key point for this technique is the choice of the angular frequency of the exciting signal. It is a trade-off between getting a high value of the modulus of the complex number, getting a stable value of this modulus in a time as short as possible, and not disturbing the process (e.g. variations of the outlet temperature of the product passing through the thermal device). To be able to make this choice, it is important to know if what has already been presented is actually happening in practice. So, the aim of the present study is to show that experimental results obtained using a specialized measurement apparatus are in a good agreement with the theoretical developments for an electrical heater.

* Corresponding author. Tel.: +33 327 511 973.
E-mail addresses: sylvain.lalot@univ-valenciennes.fr (S. Lalot), bernard.desmet@univ-valenciennes.fr (B. Desmet).

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The first part of the paper is dedicated to a brief presentation of the theory of the lock-in technique. The second part presents its application to lumped systems. The third part presents the developments for a theoretical cylindrical electrical heater in which the heat is generated by Joule effect. The fourth part is dedicated to the presentation of the test rig, including the measurement devices. The experimental results are given in the way they are presented for the theoretical electrical heater. It is completed by a comparison between experimental results and theoretical computations.

2. Basic principle of the lock-in technique

The presentation of the lock-in technique will be focused on the application to thermal devices for clarity reasons. The time signal to be analysed is supposed to be the sum of a constant value and of a purely sinusoidal signal: $e_s(t) = A_s + B_s sin(\omega_s t + \varphi_s)$. Without loss of generality, the measurement noise is considered to be a pure sinusoidal function. This noise is added to the signal to be analysed:

$$e_1(t) = e_s(t) + n(t) = A_s + B_s \sin(\omega_s t + \varphi_s) + B_n \sin(\omega_n t + \varphi_n). \quad (1)$$

This signal is multiplied by a reference signal which is also purely sinusoidal: $e_2(t) = e_r(t) = B_r sin(\omega_r t + \varphi_r)$. The result is a combination of sinusoidal functions:

$$\begin{aligned} V_2(t) = {} & A_s B_r \sin(\omega_r t + \varphi_r) + \frac{1}{2} B_r B_s \cos((\omega_r - \omega_s)t + \varphi_r - \varphi_s) \\ & - \frac{1}{2} B_r B_s \cos((\omega_r + \omega_s)t + \varphi_r + \varphi_s) + \frac{1}{2} B_r B_n \cos((\omega_r - \omega_n)t \\ & + \varphi_r - \varphi_n) - \frac{1}{2} B_r B_n \cos((\omega_r + \omega_n)t + \varphi_r + \varphi_n) \end{aligned} \quad (2)$$

Noting that the average value of any sinusoidal function over a long duration is zero, the mean value of $V_2(t)$ over a long enough time is in the specific case characterized by $\omega_r = \omega_s$,

$$\overline{V_2} = \frac{1}{2} B_r B_s \cos(\varphi_r - \varphi_s). \quad (3)$$

This mean value can be obtained either by a simple calculation or by using a low-pass filter having a sufficiently sharp transition band. When using a two-channel lock-in amplifier, a second term is computed using a 90° phase shifter applied to the reference signal. At the output of the low-pass filter of the second channel, the following value is obtained:

$$\overline{V_3} = \frac{1}{2} B_r B_s \cos\left(\varphi_r - \varphi_s - \frac{\pi}{2}\right) = \frac{1}{2} B_r B_s \sin(\varphi_r - \varphi_s). \quad (4)$$

It is possible to consider that these two values are the real and imaginary parts of a complex number Z. The modulus of this complex number is $M = 1/2 B_r B_s$, and its argument is defined by $\phi = \varphi_r - \varphi_s$. It is important to note that the preceding calculation can be extended to a more complex noise signal. As the time signal to be analyzed has a finite duration, the noise signal can be represented using a Fourier series [9]:

$$n(t) = \sum_{k=-\infty}^{k=+\infty} B_k \sin\left(2\pi k f_p t + \varphi_k\right) \quad (5)$$

where the fundamental frequency f_p is the reciprocal of the processing time, φ_k is the phase shift of each component, and B_k is the amplitude of each component. The procedure described in the case of the sinusoidal noise signal can then be used. More detailed information is available in several publications [10–12].

Note that neither the modulus nor the argument of the complex number obtained by this procedure depends on the constant value A_s.

It is also very important to note that if the signal to be analyzed, instead of being the sum of a constant value and of a pure sinusoidal function, is the sum of a linear function of time and of a pure sinusoidal function, the complex number obtained using the preceding procedure can be the same under specific conditions, as shown hereafter.

In fact, it is sufficient to show that the average value of the product of a linear function and a sinusoidal function equals zero after a long enough time under specific conditions:

$$\frac{1}{t_p} \int_{t_0}^{t_0+t_p} C_s \; t \, B_r \sin(\omega_r t + \varphi_r) dt \stackrel{?}{=} 0$$

When the processing time t_p is a multiple of the period of the reference signal and when the phase of the reference signal is linked to the angular frequency and the initial time by $cos(\omega_r t_0 + \varphi_r) = 0$ the following relations are obtained:

$$\cos(\omega_r (t_0 + t_p) + \varphi_r) = \cos(\omega_r t_0 + \varphi_r) = 0 \quad (6)$$

and

$$\begin{aligned} \int_{t_0}^{t_0+t_p} t \sin(\omega_r t + \varphi_r) dt = {} & \left[\frac{1}{\omega_r} t \cos(\omega_r t + \varphi_r)\right]_{t_0}^{t_0+t_p} \\ & + \int_{t_0}^{t_0+t_p} \frac{1}{\omega_r} \cos(\omega_r t + \varphi_r) \, dt \\ = {} & \left[\frac{1}{\omega_r} t \cos(\omega_r t + \varphi_r)\right]_{t_0}^{t_0+t_p} = 0 \end{aligned} \quad (7)$$

From a practical point of view, these criteria are not an issue. The initial time is usually zero. Hence, it is sufficient to chose $\varphi_r = 90°$, which is just an arbitrary choice. The only constraint is to use a processing time that is a multiple of the period of the reference signal.

The way to use the lock-in technique for the thermal analysis of a system is then very simple. It is necessary to excite the system in order to get an response that is similar to the afore mentioned signals. This response is then analyzed using the mathematical procedure that has been just presented. The result is an image of the state or behavior of the device. So, if there is a variation in the state of the thermal device (fouling, leakage, etc.), there will be a variation of the result. This variation is easily detected by e.g. a statistical test.

3. Theoretical applications

To facilitate the understanding of the development for the electrical heater, this part presents also the development of the calculations for a lumped system.

3.1. Lumped system

The governing equation for a lumped system in contact with a fluid (convection), having an internal source of energy $\dot{q}$, is: