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A theoretical model to predict gas permeability for slip flow through a porous medium

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HIGHLIGHTS

- A theoretical model is presented to predict the gas permeability for flow through a porous medium.
- The model maps the porous structure to a number of parallel micro-channels.
- The gas permeability is found to follow Klinkenberg's equation.
- The Klinkenberg's slip factor is obtained as a function of matrix porosity and no-slip permeability as well as gas properties.
- Results are generalized by assuming an arbitrary polygonal shape for the pores.

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ABSTRACT

Based on slip flow at pore level a theoretical model is presented to predict the gas permeability and thereby the overall pressure drop for flow through a porous medium. The model maps the porous structure to a number of parallel micro-channels of arbitrary but constant cross-sectional shapes which remains uniform along the flow path. The gas permeability is found to follow Klinkenberg's equation. The Klinkenberg's slip factor is obtained as a function of matrix porosity and no-slip permeability as well as gas properties. Results are generalized by assuming an arbitrary polygonal shape for the pores. The proposed methodology is simple to follow and easy to implement. Theoretical predictions are then compared to existing experimental data in the literature to observe good agreement.

1. Introduction

Industrial applications like electronic cooling, MEMS, fuel cells, micro-reactors, medical and biomedical devices, keep microscale transport phenomena interesting to the research community [1,2]. Such microscale transport phenomena are different from those at macroscale due to scaling effects [3,4]. In particular, owing to their small hydraulic diameters, which can be comparable to the gas molecular mean free path (MFP), the continuum assumption may break down [5,6]. Similarly, gas flow through a porous medium with pore sizes comparable to MFP experiences slip at the pore level. Here, the pores can be thought of as micro-channels with their thermo-hydraulic behaviour being different from those of macro-channels. As a result, the overall pressure drop for such porous materials is different from that of a porous medium with larger pores, e.g. the conventional Darcy flow model cannot accurately describe the gaseous flow behaviour through tight porous media. Gas flows in shales deviate from what one would expect based on Fick's and Darcy's laws. This has been of practical importance when dealing with extraction of hydrocarbon gases from unconventional gas reservoirs including coal bed methane (CBM) reservoirs. Australian industry has recently accelerated its activity in the CBM area especially in Queensland. This has brought CBM to the forefront and thereby there is a renewed interest in the problem of gas flow through tight porous media. An immediate question to answer is about the physical explanation of the difference between gas and liquid flow results. This difference has been attributed to compressibility and slip at the pore level, i.e. Klinkenberg effect [7]. According to Klinkenberg [7], gas slip flow at the solid walls of a porous medium leads to higher permeability to gas compared to that of water through the same porous medium. With pore sizes

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comparable to that of gas MFP, the frequency of collision between gas molecules and solid walls increases. This leads to an additional flux due to the gas flow at the wall surface, i.e. an increase in the gas permeability compared to that of a liquid flowing through the same porous structure. The relative increase in permeability is inversely proportional to the pore pressure with finding the proportionality constant, *b*, Klinkenberg's slip factor, being the subject of intensive research [8–28]. Tanikawa and Shimamoto [8] measured the permeability of sedimentary rocks from the Western Foothills of Taiwan, using nitrogen gas and distilled water as pore fluids to observe that gas permeability is higher than that of water by up to an order of magnitude with the difference increasing with pore pressure. The empirical power law for Klinkenberg slip factor was given by $b = 1.5K^{-0.37}$. Heid, McMahon, Nielsen and Yuster [9] used 11 synthetic cores and 164 natural cores from various oil fields in the United States to report $b \sim K^{-0.39}$. Jones [10] found that *b* generally decreases with increasing permeability according to $b \sim K^{-0.36}$ based on test results for 100 cores ranging in permeability from 0.01 to 1000 mDa. Using over 100 tight gas sand samples, Jones and Owens [11] correlated their data as $b \sim K^{-0.33}$. In a subsequent study, Jones [12] reported $b \sim K^{-0.382}$. Civan [13] modelled porous medium as a bundle of tortuous capillary tubes to come up with $b \sim K^{-0.5}$ which was close to the experimental data within an order of magnitude. However, compared to experimental results, reported in Ref. [14], i.e. $b = 0.04(K/\varepsilon)^{-053}$, the slip factor, *b*, was lower by a factor of about 4. In view of the above, the aim of this paper is to present a theoretical model to predict the pressure drop for flow through a porous medium with the pore size comparable to the gas MFP. In doing so, we start from a pore scale analysis of slip flow through a porous medium, to come up with a theoretical prediction for the gas permeability. We also report on Klinkenberg's slip factor and compare our predictions with available experimental results in the literature.

2. Analysis

A porous medium can be thought of as interconnected solid obstacles. Depending on the shape and distribution of the solid phase different flow channels can be formed and modelled. The simplest model can be constructed based on having thin parallel plates allowing for unmixed flow in the area in between the plates; see Fig. 1a. For solid spheres, for instance, a channel similar to that shown in Fig. 1b or c is formed. A general form of such channels can then be modelled as a polygon like the hexagon shown in Fig. 1d. For the purpose of this work, we assume that the flow is fully developed, unidirectional (along z) and normal to the plane. The fully developed momentum equation then takes the following form (the Poisson's equation):

$$\frac{\mathrm{d}P}{\mathrm{d}z} = \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right) \tag{1}$$

where μ is the fluid viscosity. Sparrow and Haji-Sheikh [15] offers a solution to the above equation subject to no-slip boundary condition at the wall. However, here the wall slip is important for the flow of gas through micropores where the pore size is proportional to the gas mean-free-path (MFP). This criterion can be explained mathematically knowing that the MFP is given by

$$l = \frac{k_{\mathrm{B}}T}{\sqrt{2}\pi d^2 p} \tag{2}$$

where in k_{B}, T, d and p are the Boltzmann constant, absolute temperature, gas particle diameter and pressure, respectively.

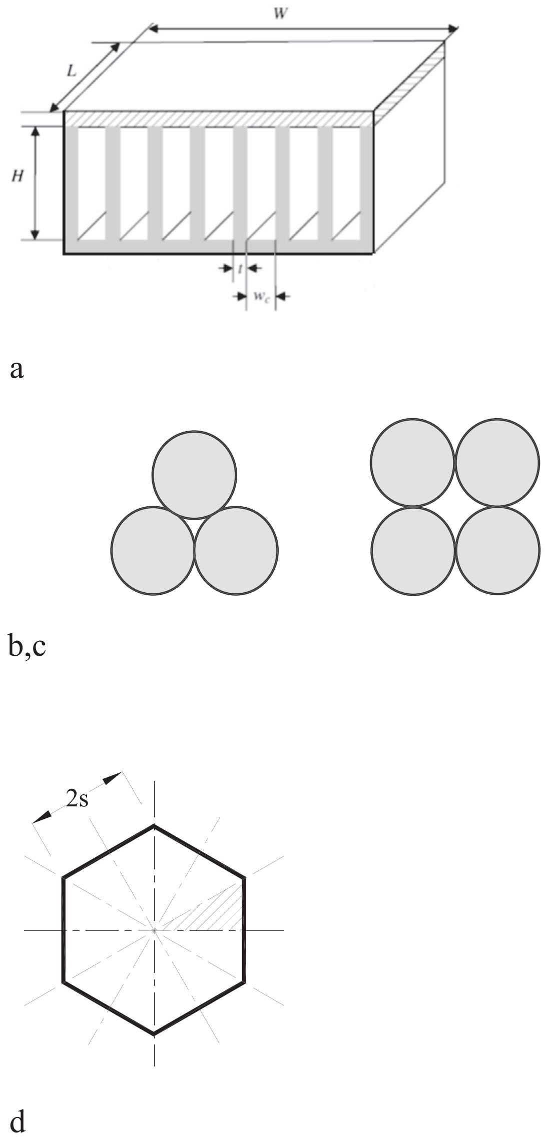

Fig. 1. a) Schematic view of a porous medium consisting of parallel rectangular channels, b) staggered arrangement of spherical particles (or cylinders), c) in line arrangement of spherical particles (or cylinders), d) the polygonal fitted to the pore formed by arbitrary solid matrix.

For slip flow (through a channel) to occur, the Knudsen number, Kn, has to be in the range 0.001 and 0.1. Lower Kn values lead to no-slip flow while for higher Kn values (than 0.1) the continuum assumption is no longer valid. Defining the Knudsen number as

$$\mathrm{Kn} = \frac{l}{D_{\mathrm{h}}} \tag{3}$$

and combining Eqns. (2) and (3) and using the pore hydraulic diameter as the characteristic length scale, to the first approximation, we have

$$\mathrm{Kn} = \frac{k_{\mathrm{B}}T}{\sqrt{2}\pi d^2 p}\frac{1}{D_{\mathrm{h}}} \tag{4}$$

This gives the criterion for the slip flow at the pore level as

$$0.001 < \frac{k_{\mathrm{B}}T}{\sqrt{2}\pi d^2 p}\frac{1}{D_{\mathrm{h}}} < 0.1 \tag{5}$$