Science and Justice 57 (2017) 80–85

Contents lists available at ScienceDirect

Science and Justice

journal homepage: www.elsevier.com/locate/scijus

Response paper

The consequences of understanding expert probability reporting as a decision☆

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ARTICLE INFO

Keywords:
Probability elicitation
Probability reporting
Decision theory
Scoring rule

ABSTRACT

In this paper we reiterate that the personalist interpretation of probability is inevitable and as least as informed as any other allegedly more 'objective' definition of probability. We also argue that the problem faced by forensic scientists, the reporting on imperfect personal knowledge, in terms of probabilities, can be reconstructed as a decision problem. Tackling this problem through a rigorous decision theoretic analysis provides further argument in support of the view that optimal probability reporting is in terms of single numbers, not intervals.

> "The calculus of probability can say absolutely nothing about reality; in the same way as reality, and all sciences concerned with it, can say nothing about the calculus of probability." [8, p. 215]

1. Introduction

So far in this collection of articles, we have argued along two main points.

> First: Investigating controversial issues regarding the likelihood ratio requires an analysis of its components. These components are conditional probabilities (or, probability densities). It is for this reason that we have called our discourse 'a question of probability, not of likelihood ratio' [5].
> Second: In essence, a probability expresses a reasoner's uncertainty about something – for example a state of nature of the past, present or future – that is not completely known to this person. For the purpose of the discussion here, it is also common to refer to uncertain quantities in terms of propositions (e.g., the proposition that the analytical features of a crime stain are of type Γ).

According to the above, a probability is one's expression of uncertainty about an unknown[1] quantity or state of nature, but one is not uncertain about one's probability. It is unsound, thus, for a person to make statements such as:

> 'I am unsure about the probability', or 'the probability is unknown (to me)'.

On that account, probability is not something that exists in the real-world that surrounds us, independently of an individual mind that contemplates about a particular aspect of the world. By extension, ratios of probabilities, too, do not exist, as noted in [3].

The measure of one's uncertainty about an unknown quantity or state of nature is a single number – a probability (yours, ours, anybodies) – for as different numbers, by definition, express different states of uncertainty. And, to emphasize this once again, the notion of uncertainty does *not* relate to the numerical probability that each and every person detains in their own way. Uncertainty relates to a proposition, the truth of which may be under dispute, and probability, in terms of a number, is the expression of an individual's personal state of uncertainty, about the proposition of interest.

☆ This paper is part of the Virtual Special Issue entitled: Measuring and Reporting the Precision of Forensic Likelihood Ratios, [http://www.sciencedirect.com/science/journal/13550306/vsi], Guest Edited by G. S. Morrison.

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[1] It is important to emphasize that the imperfect knowledge intimately relates to the person who expresses a probability. It may well be that another person has a more elaborate knowledge base, or even complete knowledge about the particular state of nature or quantity of interest.

http://dx.doi.org/10.1016/j.scijus.2016.10.005

Reporting a likelihood ratio as an interval would amount to reporting probability intervals for the numerator and the denominator: but is this the proper understanding of the notion of interval? Part of basic understanding is that probability is distributed over the various outcomes that an uncertain discrete quantity (e.g., the number of GSR particles) may take. In the same way, in presence of an uncertain continuous quantity (e.g., a population proportion θ), one may consider how much probability is assigned to particular ranges of possible values of the random quantity. For example, one may consider one's probability that an unknown population proportion θ lies between 0.6 and 0.8. So, there is an interval here, but it relates to the uncertain quantity θ, and *not* to the probability one specifies for the interval of values of θ. There is uncertainty about the proportion θ lying in the interval between 0.6 and 0.8. Probability expresses this uncertainty with a number, but here is no interval about this numerical probability. From this it follows that, since there is no interval (or, uncertainty) around a probability, there is also none for a ratio of two probabilities (i.e., the likelihood ratio).

Two aspects of the above starting point continue to raise discussion among some forensic scientists, as is demonstrated by position papers published so far in this Special Issue, but also in legal literature at large. The first aspect concerns the understanding that probability is an expression of personal belief[2] of an individual about something that is uncertain to this individual. Some commentators view this interpretation of probability skeptically and criticize it as being inappropriate. The second aspect concerns the understanding of probabilities, and hence likelihood ratios, being single numbers. Some quarters argue that this provides a poor descriptive account of how people intuitively perceive uncertainty and hence consider this to be a perspective difficult to adopt in practice. In this paper, we will discuss these two aspects in turn, and outline the reasons why we disagree with these skepticisms.

The paper is structured as follows. In Section 2, we will take a closer look at the notion of probability as personal degree of belief and discuss what this view of probability means and does not mean. We will insist on the point that the term 'personal' associated with the belief type interpretation of probability is not a synonym for arbitrary and speculation, and hence does not render personal probability inappropriate for forensic science. In Section 3, we will provide further argument — not raised so far in this Special Issue — in support of the view that probability is given by a single number. We will do this by introducing the notion of decision. Specifically, we will use decision theory as the overarching conceptual and analytical framework. Starting from only a few basic assumptions, we will engage in a defensible series of operations to derive all positions that we highlight in Sections 2 and 3: belief type probability and probability as a single number. In our discussion and conclusion, Section 4, we will emphasize that our formal approach to inference and decision is of normative nature. It precedes and is to be distinguished from empirical and descriptive accounts. Trying to bend the normative approach in order to satisfy descriptive criteria and the intuitive perception of human inference and decision behavior would be a misunderstanding.

2. Subjective (personal) probability: what it means and what it does not mean

The belief type interpretation of probability is retained here because other interpretations, such as the frequentist definition, involve assumptions that are known to fall short of the features of the real-world applications they should be able to capture. Frequentist ideas involve the notion of repetition under stable conditions, and the counting of the number of times a particular outcome occurs. This includes extensions to idealisations, such as long run repetitions in the context of an infinitely repeatable experiment. Although this perspective may have some appeal for classroom experiments and artificial conditions (e.g., flipping coins or rolling dice[3]), it readily reaches its limits with real-world situations that are highly distinctive and non-repeatable. This leads to applicability problems that continue to frustrate generations of practitioners, yet the frequentist approach continues to be the predominant perspective taught in basic science education. This is all the more surprising given that there is an alternative — the belief type interpretation — that is capable to cope with the features of real world events. What is more, paradoxically, it is fraught with prejudice.

The belief type interpretation differs from the frequentist viewpoint in two main respects. First, the belief type interpretation of probability does not require that a target event (or, experiment) be repeatable. Second, probability is not seen as a property of the real-world — also sometimes called system — under observation. Instead, probability is considered as a property of the person who contemplates about the real-world. For example, when we consider the truth or otherwise of a proposition regarding, for example, the outcome of an experiment (e.g., the comparison of DNA profiles of questioned and items of known origin), the frequentist might say that in his view the probability of the event of encountering corresponding DNA profiles is the long-run relative frequency of this outcome in the experiment under investigation, but that he does not know this value (relative frequency). So, the frequentist would express himself in terms of 'not knowing the probability', or 'unknown probability'. Taking this answer literally — probabilities being unavailable — leads to the conclusion that the scientist cannot offer help with the problem of interest, because the use of a likelihood ratio in forensic science requires him to be able to specify probabilities.

In the belief type interpretation of probability, the above impasse does not occur. Indeed, when considering probability in terms of a person's belief, it is meaningless to say that probability is not known to that person. A person necessarily knows what she thinks or else should not be considered entitled to talk sensibly about the uncertain proposition of interest. But notice that the knowledge of two persons regarding the truth or otherwise of a given proposition may differ, and sometimes substantially so, which will result in them assigning different probabilities. So differences in assigned probabilities are not surprising, they merely reflect the capacity of the framework to account for inter-individual differences in personal knowledge.

The above does not mean, however, that the adherents of personal probability may not consider, too, data from repeated trials where they are available. As noted by De Finetti [9, p. 334]:

> "Those interpretations of the notion of probability in a (would-be) objective sense which are based on symmetry (the classical conception; equally likely cases), or on frequency (the statistical conception; repeated trials of a phenomenon), provide criteria which are also accepted and applied by subjectivists (…). It is not a question of rejecting them, or of doing without them; the difference lies in showing explicitly how they always need to be integrated into a subjective judgment, and how they turn out to be (more or less directly) applicable in particular situations. If one, instead, attempts to force this one or that one into the definitions, or into the axioms, one obtains a distorted, one-sided, hybrid structure."

Clarification of this point can also be found in the writings of Lindley [e.g., 13] who prefers to keep the concept of relative

[2] Throughout this paper, we will take 'belief type', 'personal' and 'subjective' as referring to the same interpretation of probability.

[3] Note however that even for this kind of experiment, the assumption of repetitions under stable conditions cannot be uphold, as with increasing numbers of repetitions, the coins and dice may wear out.