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Original Research Paper

Simulation of particle-laden flow in a Humphrey spiral concentrator using dust-liquid smoothed particle hydrodynamics

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ABSTRACT

This paper demonstrates that our extended smoothed particle hydrodynamics (SPH) model can successfully simulate multiphase flow in a Humphrey spiral concentrator (HSC) with two phases: powder and water. The powder phase in the model was assumed to be a continuum, as the spacing between particles in this state is much smaller than the typical length scale of flow. Further investigation was conducted on the influences of various design factors of the HSC, including the descent angle and curvature profile of the trough, during the separation of a binary mineral particle mixture.

The model was validated by comparing the simulated results with the experimental results of Loveday and Cilliers (1994) as well as those of a novel lab-scale-experiment using a miniature of the HSC. The proposed SPH model accurately simulated dusty liquid flow in the HSC in both cases with an acceptable degree of accuracy relative to the experimental results. These studies are expected to be useful in future optimizations of HSC design and operating conditions.

1. Introduction

The Humphrey spiral concentrator (HSC) is a widely used mineral separator that sorts particles on the basis of differences in the specific gravities of particles. Since its invention by Ira. B. Humphrey in the 1940s, HSCs have been used over decades in both mineral and coal industry, given their advantages such as low cost, long life, and good recovery.

An HSC consists of an open trough that twists downward helically about a central axis. In the HSC, separation of gangue particles from pure mineral is achieved by a combined action of gravity and hydrodynamic forces owing to a circulating flowing film. Despite the simple structure of the equipment, the separation mechanism of the HSC is complicated and difficult to be explained by any simple mathematical descriptor.

Numerous empirical models have been developed for spiral concentrators on the basis of experimental data [1–8]. Some of these models have been developed according to simple force balances to calculate the equilibrium position of particles. However, the major drawback of these models is that the separation properties are characterized by some empirical parameters and extensive tests are required to obtain these parameters under changing material type, spiral type, or particle size range.

To overcome this drawback, a few works performed mechanistic analysis based on computational fluid dynamics (CFD) [9–14] or the discrete element method (DEM) [15] to model the flow in an HSC. Despite the partial successes of these models, they did not satisfactorily take into account the geometric complexity of the flow within an HSC. The profiles of free surfaces are mutually related to the structure of secondary circulation, and they therefore play a significant role in the separation. The interaction between flow and particulate matters (hereafter referred to as "dust") can change the flow profile in a rather complicated manner. Owing to the intrinsic geometric complexity of the flow in an HSC, modeling of this flow by using a grid-based Eulerian approach is difficult. An adaptive-grid technique [12] or volume-of-fluid [9–11] method have been employed in previous Eulerian works with moderate success in overcoming this problem.

Smoothed particle hydrodynamics (SPH) is a particle-based Lagrangian CFD model. It was originally proposed for simulating nonaxisymmetric astrophysical phenomena [16,17]; however, its application has been broadened to various areas of fluid-related computational physics. One of the major advantages of SPH over the conventional grid-based CFD is that SPH bridges the gap between continuum approximation and discontinuous fragmentation in a natural way owing to its Lagrangian property. A geomet-

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rically complicated flow including free surfaces can therefore be easily modeled without any severe conflict, which is inevitable in a conventional grid-based method.

The formulation of SPH for dusty gas was first described by Monaghan and Kocharyan [18]. This model assumes that the interspacing between dusts is sufficiently smaller than the typical length scale of the flow, and therefore, the dust phase can be treated as a continuum. Recently, this model was generalized to simulate dusty liquids for applications to the sedimentation of layers of particles in a static tank [19] and in a turbulent medium [20]. The results of the model agreed well with experimental and theoretical results. Irrespective of whether powders are buoyant or sinking in homogeneous or stratified fluids or sedimenting in a turbulent fluid, the powders can be simulated efficiently and accurately by using the model. The model was also recently improved by Kwon and Cho [21], who proposed a novel consecutive approach for computing the pressure interaction term between liquids and dust.

In the present work, we demonstrate that our dusty-liquid SPH model can successfully simulate the flow and particle behavior within an HSC. We first describe the governing equations of the SPH model, which is extended to describe dusty liquid flow, and then, we present the setup conditions employed for the simulation of flows in an HSC. Next, we demonstrate the influence of various design factors, such as the vertical traveling distance, descent angle, and curvature profile of the trough, on the particle separation. Finally, the results of two validation tests are presented alongside a comparison of the simulated results to the experimental results of Loveday and Cilliers [1] and a lab-scale test using a miniaturized HSC.

2. Model description

Although the essentials of the governing equations for dust-liquid multiphase flow are described in detail in the present paper, readers should refer to Kwon and Monaghan [19], Kwon and Monaghan [20] for further details of these equations. Moreover, the basic approximation methodology of standard SPH is well described in Monaghan [22].

2.1. Continuum equations of motion

The equations for continuum analysis of dusty fluid are those given by Harlow and Amsden [23,24]. The equations of conservation of mass and acceleration for dust-liquid multiphase flow are given by

$$\frac{d\hat{\rho}_l}{dt} = -\hat{\rho}_l \nabla \cdot \mathbf{v}_l \tag{1}$$

$$\frac{d\hat{\rho}_d}{dt} = -\hat{\rho}_d \nabla \cdot \mathbf{v}_d \tag{2}$$

$$\frac{d\mathbf{v}_l}{dt} = -\frac{\nabla P}{\rho_l} - \frac{K}{\hat{\rho}_l}(\mathbf{v}_l - \mathbf{v}_d) + \mathbf{g} \tag{3}$$

$$\frac{d\mathbf{v}_d}{dt} = -\frac{\nabla P}{\rho_d} - \frac{K}{\hat{\rho}_d}(\mathbf{v}_d - \mathbf{v}_l) + \mathbf{g} \tag{4}$$

where subscripts l and d refer to liquid and dust, respectively; P is the pressure; and K is a drag factor that depends on the local properties of the dust and liquid. Further, $\mathbf{v}_l$ and $\mathbf{v}_d$ are the liquid and dust velocities, respectively; $\mathbf{g}$ is the external self-gravity. $\hat{\rho}_l$ and $\hat{\rho}_d$ are the volume densities of the liquid and dust, respectively. These densities are related to the real material densities of each phase ρ_l and ρ_d, by

$$\hat{\rho}_l = \theta_l \rho_l \tag{5}$$

$$\hat{\rho}_d = \theta_d \rho_d \tag{6}$$

where θ_l and θ_d are the volume fractions of liquid and dust, respectively, and they therefore satisfy the following condition:

$$\theta_l + \theta_d = 1 \tag{7}$$

2.2. SPH equations of motion

In SPH, the particles are, in principle, mathematical interpolation points that are used to solve the governing equations; however, they also represent physical material. The concept of SPH dust particles needs to be understood in the same manner. In the first point of view, SPH dust particles are used to calculate properties such as the volume density of the dust phase. In the second point of view, an SPH dust particle represents an assembly of physical dust particles that—depending on their mass, size, and number density—could be several hundred thousand in number. The mass of an SPH dust particle is the mass of this assembly of physical dust particles. The local interaction between the two phases can then be interpreted numerically from the interaction between SPH liquid particles and SPH dust particles. For example, the drag between the liquid and the physical dust particles is the drag between the assembly of physical dust particles, represented by the SPH dust particles, and the liquid.

Hereafter, we use the convention that subscripts a and b represent the liquid particles and subscripts i and j represent the dust particles. The SPH forms of Eqs. (1) and (2) for particles a and i are then given as follows:

$$\frac{d\hat{\rho}_a}{dt} = \sum_b m_b \mathbf{v}_{ab} \cdot \nabla_a W_{ab}(\bar{h}_{ab}) \tag{8}$$

$$\frac{d\hat{\rho}_i}{dt} = \sum_j m_j \mathbf{v}_{ij} \cdot \nabla_i W_{ij}(\bar{h}_{ij}) \tag{9}$$

where for example, $m_b\psi$ is the mass of particle b, $\mathbf{v}_{ab} = \mathbf{v}_a - \mathbf{v}_b$, and $\bar{h}_{ab} = (h_a + h_b)/2$. $W(h)$ is the smoothing kernel that is normalized to 1; it converges to the Dirac delta function as the smoothing length h approaches 0. $W_{ab}(h) = W(|\mathbf{r_a} - \mathbf{r_b}|, h)$, and ∇_a denotes the gradient with respect to the coordinates of particle a. The liquid acceleration equations are as follows:

$$\frac{d\mathbf{v_a}}{dt} = \phi_a + \Psi_a - \sum_b \Pi_{ab} m_b \nabla_a W_{ab}(\bar{h}_{ab}) - \sigma \sum_j m_j \frac{K_{aj}}{\hat{\rho}_a \hat{\rho}_j}(\mathbf{v}_{aj} \cdot \hat{\mathbf{r}}_{aj})\hat{\mathbf{r}}_{aj} W^{\mathcal{D}}_{aj}(h_a) + \mathbf{g}, \tag{10}$$

where

$$\phi_a = -\sum_b \left(\frac{\theta_a P_a}{\hat{\rho}_a^2} + \frac{\theta_b P_b}{\hat{\rho}_b^2} \right) m_b \nabla_a W_{ab}(\bar{h}_{ab}). \tag{11}$$

and they take the following form for dust:

$$\frac{d\mathbf{v}_i}{dt} = -\sum_j m_j \left(\frac{\theta_i P_i + \theta_j P_j}{\hat{\rho}_i \hat{\rho}_j} \right) \nabla_i W_{ij}(\bar{h}_{ij}) + \Psi_i - \sigma \sum_b m_b \frac{K_{ib}}{\hat{\rho}_i \hat{\rho}_b}(\mathbf{v}_{ib} \cdot \hat{\mathbf{r}}_{ib})\hat{\mathbf{r}}_{ib} W^{+}_{ib}(h_b) + \mathbf{g}, \tag{12}$$

where $\mathbf{r}_{aj} = \mathbf{r}_a - \mathbf{r}_j$ and $\hat{\mathbf{r}}_{aj} = \mathbf{r}_{aj}/|\mathbf{r}_{aj}|$. σ is a constant whose value is equal to the dimension number. It should be noted that the volume fraction θ is expressed in accordance with the corresponding particle phase. The kernels W and $W^{\mathcal{D}}$ are, respectively, given by

$$W(q) = \frac{21}{256\pi}(1 + 2q)(2 - q)^4 \tag{13}$$