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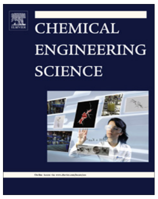

Modeling droplet dispersion in a vertical turbulent tubing flow

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HIGHLIGHTS

- A model of droplet dispersion in a vertical production tubing is developed.
- An advection-diffusion-population balance equation is solved.
- Model performance is illustrated by computations of water in oil dispersion.

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ABSTRACT

Usually, during oil production, water and oil flow simultaneously in the wellbore. When water holdup in the borehole is small, water droplets may be dispersed into bulk oil making water breakthrough detection a challenging task. In this paper, a comprehensive engineering model of droplet dispersion is presented.

Dispersion of droplets in a long vertical turbulent tubing flow is modeled by an Advection-Diffusion-Population Balance equation. The Prandtl Mixing-Length model of turbulence is used to describe the velocity profile across a tubing. The turbulence energy dissipation rate distribution across a pipe is calculated by an analytical equation. The fixed pivot method is employed for calculation of the population balance term of the governing equation. Droplet fragmentation is modeled using a recently developed droplet breakup model (Eskin et al., 2017). It is assumed that volume concentration of a dispersed phase does not exceed 10%. A computational code developed allows tracking evolution of droplet size distribution along a tubing. Model performance is illustrated by computations of the water in oil dispersion process. Effects of oil/water interfacial tension, well production rate and oil viscosity on dispersion are demonstrated.

1. Introduction

The most obvious and practically important example of dispersion of immiscible liquids in a turbulent pipe flow is related to crude oil-water mixture flow in a production tubing (e.g., Ahmad et al., 2013). Usually, water droplets are dispersed in oil. Formation of a stable water-in-oil emulsion (i.e., one in which droplets do not coalesce) in a production tubing is a highly undesirable phenomenon. Natural surfactants (e.g., asphaltenes and carboxylic acids), which are present in crude oil, cause emulsion stabilization during oil production. Droplets comprising a stabilized emulsion are intensely fragmented in a turbulent flow and can reach very small sizes downstream. A stable emulsion flow is characterized by enhanced friction losses, complicates oil/water separation as well as some measurements associated with detection of separate droplets in a flow.

In general, if formation of a stable emulsion is expected, then an injection of demulsifying chemicals may be needed for emulsification prevention. If concentration of water droplets is low, generation of a stable emulsion does not lead to a significant friction losses. Nevertheless, even in this case, emulsion composed of very small droplets can cause serious problems. For example, some production logging tools measure gas, oil and water holdup on the basis of differentiation of electric conductivities. However, measurement devices of such a type cannot detect droplets, which are smaller than a certain threshold. The detection limit, defined as the critical droplet size, is typically in the range of $d_{cr} \sim 200$–$300\ \mu m$. The ratio of volume concentration of droplets with sizes below the critical size to the total droplet volume concentration, averaged over a pipe cross-section, is equal to the measurement error. Forecasting size distribution of droplets entering a

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Nomenclature

c_D	droplet drag coefficient
D	tubing diameter, m
D_i	turbulent diffusivity of i-th size fraction droplet, m^2/s
f	Fanning friction factor
f_{bv}	breakup volume fraction
Fq	turbulent eddy fluctuation frequency, 1/s
G	droplet breakup rate, 1/s
N	number concentration of droplets per unit volume, $1/m^3$
$P(\upsilon)$	probability of droplet breakup at interaction with a turbulent eddy
q	droplet volume flux, m/s
r	radial coordinate, m
Re	tubing Reynolds number
Re_i	droplet Reynolds number
Sc_t	turbulence Schmidt number
Sc_{pt}	turbulence Schmidt number for a particle
s	ρ_d/ρ_f
T_L	Lagrangian time scale, s
u	axial flow velocity, m/s
u_{si}	fluid-droplet relative velocity (settling velocity) of the i-size fraction, m/s
u_*	friction velocity, m/s
U	mean flow velocity, m/s
We	Weber number
y	coordinate determining distance from the wall, m

Greek symbols

β	droplet breakup density function
δ	viscous sublayer thickness
ε	turbulence energy dissipation rate per unit mass, W/kg
ϕ	dispersed phase volume fraction
γ	interfacial tension, N/m
κ	von Kármán constant
λ_0	inner (Kolmogoroff) turbulence scale, m
μ	dynamic viscosity, Pa s
v	kinematic fluid viscosity, m^2/s
ρ	fluid density, kg/m^3
τ	shear stress, Pa
τ_i	relaxation time of a droplet of the i-th size fraction, s
τ_w	wall shear stress, Pa
υ	droplet volume, m^3

Subscripts

cr	critical
d	droplet
f	continuous fluid
i, j, k	sequential numbers of droplet size fractions
t	turbulent
0	initial

production logging tool would allow altering (moving upstream) its position in a production tubing to provide an acceptable measurement error.

Note, the tendency of a hydrocarbon mixture to form a stable emulsion can be determined by a simple emulsion stability test applied to a mixture sample under pressure and temperature corresponding to downhole conditions. Frequently, an emulsion formed is not completely stable, although the droplet coalescence rate is significantly reduced. Modeling of such a partially stable emulsion is an extremely complicated problem and beyond the scope of the present work. In practical applications, a partially stable emulsion can be modeled assuming that it is completely stable. A computational result, in that case, corresponds to a useful conservative (worst case) scenario. Droplet size distribution in a pipeline flow strongly depends on oil/water interfacial tension, flow rate, oil viscosity and a distance along a pipeline determining residence time during which droplets are being dispersed.

Modeling droplet dispersion in a pipe flow requires mathematical description of the following phenomena: (1) turbulent flow field; (2) droplet transport; (3) evolution of size distribution of droplets due to their breakups.

In the current work, we will limit our modeling to vertical pipes to exclude accounting for droplet stratification across a tubing due to gravity. Note, in practice, vertical wells represent a significant fraction of all oil producing wells. The dispersion process can be confidently assumed to be steady-state; i.e., droplet size distribution changes only along and across a tubing. We realize that commercial CFD codes (e.g., ANSYS Fluent, STARCCM+) could be used for modeling such a problem. At first glance, using a commercial code is computationally expensive because tubing length can reach hundreds of meters. Nevertheless, CFD computations of the steady-state dispersion process in a long pipe can be handled using a short pipe section with periodic boundary conditions. However, running a commercial CFD code under field conditions is still impractical and a separate code suitable for running on a laptop would be preferable. Therefore, the objective of this research is development of an engineering model of droplet dispersion and a corresponding computational code, whose accuracy is sufficient for field applications.

In the present work, we will limit the dispersed phase volume concentration to 10%, i.e., at early water breakthrough, that may correspond to a significant part of real field cases. Currently, a reliable droplet breakup model, valid for a high dispersed phase concentration, is unavailable. Note, a frequently used breakup model correction on high droplet concentration, proposed by Coulaloglou and Tavlarides (1977), has no strong physical background and never been carefully validated. For low droplet concentrations, an effect of droplets on a continuous fluid flow is negligible and a steady-state turbulent pipe flow can be accurately described by simple relations based on the boundary layer approach (e.g., Schlichting and Gersten, 2000). This simple flow model will be a basis for a dispersion model presented further.

There are a few papers dedicated to investigation of droplet dispersion in a turbulent pipe flow. However, those works are mainly reduced to measurements and/or predictions of the maximum droplet size under steady-state conditions (e.g., Hesketh et al., 1987; Boxall et al., 2012; Eskin, 2015). We would like to specifically mention the work of Simmons and Azzopardi (2001) who measured mean diameters of coalescing dispersions in both horizontal and vertical pipe sections at certain points of a flow loop. The measured mean droplet diameters were compared with those calculated by different known correlations.

Let us briefly discuss the work of Kostoglou and Karabelas (2007) who solved a problem that is close to that considered here. The authors modeled dispersion of non-coalescing droplets in a steady-state turbulent pipe flow assuming gravity was negligible. They suggested using an advection-diffusion-population balance equation for this purpose. However, these authors did not include the population balance term in the model, instead assuming that although droplets are fragmented they are locally monodispersed whereas their diameters vary across and along a pipe. We would like to mention also that Kostoglou and Karabelas (2007) did not