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Improvement of the two-fluid momentum equation using a modified Reynolds stress model for horizontal turbulent bubbly flows

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HIGHLIGHTS

- Numerical simulations are performed for horizontal turbulent bubbly flows.
- Three different ways to formulate the momentum diffusion terms are tested.
- Proper forms of the momentum diffusion terms are suggested.

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ABSTRACT

Two-fluid equations are widely used for practical applications involving multi-phase flows in chemical reactor, nuclear reactor, desalination systems, boilers, and internal combustion engines. The popular two-fluid equation for a gas-liquid two-phase flow is based on the assumption of interpenetrating continua. According to the experimental data of fully-developed turbulent bubbly flows in a horizontal pipe, the bubble phase velocity is close to or slightly smaller than the liquid phase velocity. The velocity profile is nearly symmetric along the vertical centerline of the horizontal pipe, or tends to be slightly skewed toward the bottom region of the pipe. However, numerical simulations using the momentum equation based on interpenetrating continua showed that, in contrast, the bubble phase was faster than the liquid phase. In addition, the velocity profile was predicted to be skewed toward the upper region of the pipe. These simulation results are not consistent with experimental observations. In the meantime, there are particle averaged momentum equations in which the continuous and disperse phase equations are developed from the equations of motions of fluid and particle, respectively. We considered two different particle averaged momentum equations. The form of one particle averaged momentum equation is similar to that of the momentum equation based on interpenetrating continua, except for the laminar viscosity term. Thus, for a turbulent bubbly flow, this particle averaged equation showed similar results as observed in the momentum equation based on interpenetrating continua. The other particle averaged equation differs from the momentum equation based on interpenetrating continua in both laminar and turbulent viscosity terms. This particle averaged equation showed good agreement with experimental observations.

1. Introduction

Two-fluid equations are widely used for practical applications involving multi-phase flows in chemical reactor, nuclear reactor, desalination systems, boilers, and internal combustion engines. There is no doubt that the two-fluid equations are an extremely useful tool to predict the macroscopic behavior of a multi-phase flow.

The popular two-fluid equations are obtained by applying the time- or volume-averaging method to the local instantaneous conservation equations (Drew, 1983; Ishii, 1975). All phases are assumed to be continuous fields (interpenetrating continua), and the same averaging process is applied to all phases, irrespective of the phase topology. For an adiabatic two-phase flow, the volume-averaged momentum equation for phase k can be written in the form

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$$\frac{\partial}{\partial t}(\alpha_k\rho_k\mathbf{u}_k) + \nabla\cdot(\alpha_k\rho_k\mathbf{u}_k\mathbf{u}_k) = -\alpha_k\nabla p_k + \nabla\cdot(\alpha_k\boldsymbol{\tau}_k) + \nabla\cdot(\alpha_k\boldsymbol{\tau}_k^{Re}) + \alpha_k\rho_k\mathbf{g} + \mathbf{f}_{ik} + (p_{k,i} - p_k)\nabla\alpha_k - \boldsymbol{\tau}_{k,i}\cdot\nabla\alpha_k, \quad (1)$$

where α_k, ρ_k, $\mathbf{u}_k$, p_k, $\boldsymbol{\tau}_k$, $\boldsymbol{\tau}_k^{Re}$, $\mathbf{g}$, $\mathbf{f}_{ik}$, $p_{k,i}$, and $\boldsymbol{\tau}_{k,i}$ are the phase fraction, density, velocity vector, pressure, viscous stress tensor, Reynolds stress tensor, gravitational acceleration, and generalized interfacial drag, average interfacial pressure, and average interfacial viscous stress tensor, respectively. The last two terms in Eq. (1) can be important for separated flows (Ishii and Hibiki, 2011; Ishii and Mishima, 1984), but can be neglected for other flows. As a result, for a dispersed flow, the following equation is used:

$$\frac{\partial}{\partial t}(\alpha_k\rho_k\mathbf{u}_k) + \nabla\cdot(\alpha_k\rho_k\mathbf{u}_k\mathbf{u}_k) = -\alpha_k\nabla p_k + \nabla\cdot(\alpha_k\boldsymbol{\tau}_k) + \nabla\cdot(\alpha_k\boldsymbol{\tau}_k^{Re}) + \alpha_k\rho_k\mathbf{g} + \mathbf{f}_{ik}. \quad (2)$$

However, the above equation yields a physically incorrect result. Consider a two-phase flow at rest without gravity. Then, the last two terms of Eq. (1) induce a source of momentum solely due to the spatial arrangement of the phase (Harlow and Amsden, 1975). To avoid such a unphysical situation, Prosperetti (2007) suggested

$$\frac{\partial}{\partial t}(\alpha_k\rho_k\mathbf{u}_k) + \nabla\cdot(\alpha_k\rho_k\mathbf{u}_k\mathbf{u}_k) = -\alpha_k\nabla p_k + \alpha_k\nabla\cdot\boldsymbol{\tau}_k + \nabla\cdot(\alpha_k\boldsymbol{\tau}_k^{Re}) + \alpha_k\rho_k\mathbf{g} + \mathbf{f}_{ik}. \quad (3)$$

In the meantime, there is another type of momentum equation, that is, particle averaged equation. The averaged momentum equations for the dispersed and continuous phases are obtained by applying the local averaging process to the equation of motion for the center of mass of a single particle and the Navier-Stokes equation, respectively (Anderson and Jackson, 1967; Crowe et al., 2011; Prosperetti and Jones, 1984). This particle averaged equation has been usually used for a gas-solid particle system; however it can also be used for a gas-liquid system (Lee et al., 2017; Moraga et al., 2006; Zhang and Prosperetti, 1994). Unless the interaction between the fluid particles is considered, the main difference from Eq. (2) is in the momentum diffusion terms. According to Anderson and Jackson (1967), $\alpha_k\nabla\cdot\boldsymbol{\tau}_c + \alpha_k\nabla\cdot\boldsymbol{\tau}_c^{Re}$ is substituted for the second and third terms on the right of Eq. (2), where the subscript c signifies the continuous phase. On the other hand, Crowe et al. (2011) formulated the momentum diffusion terms as $\alpha_k\nabla\cdot\boldsymbol{\tau}_c + \nabla\cdot(\alpha_k\boldsymbol{\tau}_k^{Re})$. In the particle averaged momentum equation, the viscous stress term is written as $\alpha_k\nabla\cdot\boldsymbol{\tau}_c$. However, the Reynolds stress term varies depending on literature. Though the differences in the momentum equations are seemingly due to the formally different averaging, the differences may be due to some looseness of unclosed terms.van Wachem et al. (2001) noticed a difference between the momentum equation based on interpenetrating continua (Ishii, 1975) and the particle averaged equation (Anderson and Jackson, 1967), and performed a comparative study on the momentum equations in a vertical gas-solid system. The predictions based on the two different momentum equations did not differ in terms of the macroscopic flow behavior; however, the flow of the gas phase was slightly different in areas with a large solid fraction gradient. That result is attributed to the fact that the turbulence effect was not considered in the simulations.

We now discuss the relative velocity between the two phases. Podowski (2009) mentioned the possibility that the dispersed phase can be predicted to be faster than the continuous phase even for a fully-developed dispersed flow, unless the wall drag force on the dispersed phase is properly considered in the one-dimensional averaged momentum equation. Is it possible for the bubble phase to be faster than the liquid phase in a fully-developed bubbly flow? Bottin et al. (2014) measured the bubble and liquid velocities in a short horizontal pipe. Fig. 1 shows the bubble and liquid velocity profiles along the vertical centerline from the bottom wall to the top wall. One can see that the bubble phase is slightly slower than the liquid phase. Kong and Kim (2017) and Talley et al. (2015) measured bubble velocities in a long horizontal pipe, and compared the result with the liquid-phase velocity estimated using the approach employed by Kocamustafaogullari and Huang (1994); the bubble phase was slower than the liquid phase (not shown here). It is obvious that the dispersed-phase is not faster than the continuous phase for a fully-developed dispersed flow.

Recently, Lee et al. (2017) applied the particle averaged equations, which have been used for solid-gas flows, to bubbly flows. Lee et al. (2017) performed numerical simulations for multi-dimensional bubbly flows under a conceptual condition in which the effects of gravity, lift force, wall lubrication, and turbulent dispersion are excluded. For such an ideal condition, the bubble phase experiences only the interfacial drag caused by the relative velocity between the two phases, and the velocities of the two phases are expected to equalize for a fully-developed bubbly flow in a horizontal pipe. However, the momentum equation based on interpenetrating continua (Eq. (2)) predicted that the bubble phase was faster than the liquid phase. Even when $\alpha_k\nabla\cdot\boldsymbol{\tau}_c + \nabla\cdot(\alpha_k\boldsymbol{\tau}_k^{Re})$ was used for the momentum diffusion terms, the bubble phase was predicted to be faster than the liquid phase if a flow was turbulent. On the other hand, when $\alpha_k\nabla\cdot\boldsymbol{\tau}_c + \alpha_k\nabla\cdot\boldsymbol{\tau}_c^{Re}$ was substituted for the momentum diffusion terms, the two phase velocities were equalized for both the laminar and turbulent flows.

So far, we have mentioned different momentum equations. This study presents numerical simulations for fully-developed horizontal bubbly flows to examine the validity of the two-fluid momentum equations. While Lee et al. (2017) considered ideal conditions, this study considered real experimental conditions. For horizontal flows, the difference between the stream-wise velocities of the two phases can be used to examine the validity of each momentum equation. There are previous numerical simulations of horizontal bubbly flows using the momentum equation based on interpenetrating continua (Ekambara et al., 2008; Li et al., 2010; Yeoh et al., 2012) or the mixture equation (Shang et al., 2013). The previous works showed reasonable predictions for Kocamustafaogullari's experimental data (Kocamustafaogullari and Huang, 1994; Kocamustafaogullari et al., 1994; Kocamustafaogullari and Wang, 1991). However, the

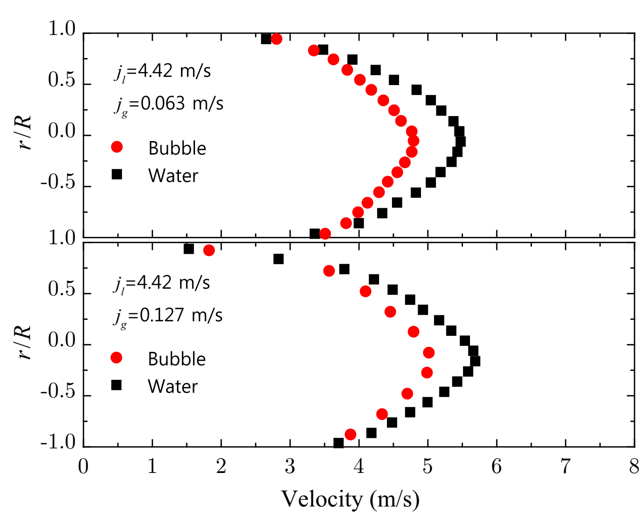

Fig. 1. Bubble and water velocity profiles along the vertical centerline in a horizontal pipe (Bottin et al., 2014).