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Numerical simulation of two-phase flow in porous media using a wavelet based phase-field method

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HIGHLIGHTS

- The capillary phenomena have been studied based on the free-energy of multiphase flow.
- A wavelet-based phase-field method for multiphase flow has been investigated.
- Bubble dynamics has been compared with an experiment and a reference numerical model.
- Bubble velocity predicted by the phase-field method agrees with experimental data.
- Results indicate that phase-field calculations are as accurate as some experiments.

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ABSTRACT

An understanding of the transport and dynamics of two fluids in porous media, as well as the bubbly flow regime, is important for many engineering applications, such as enhanced oil recovery (EOR) method, drilling technology, multiphase production system, etc. In this respect, the dependence of capillary stresses on the excess free-energy of a thin interfacial layer formed by two immiscible fluids is not fully clear, particularly in porous media. Of particular interests are the closure models for interphase forces which often hinder the reliable prediction of the homogeneous flow regime. This article presents a multiphase Computational Fluid Dynamics (CFD) study of bubbles in homogeneous porous media to model the flow of oil and gas, and investigates a closure model that is based on the Allen-Cahn phase-field method, where the capillary stress is derived from the excess free-energy. The governing dynamics is simulated with the volume averaged Navier-Stokes equations extended for multiphase flow in porous media. The equations have been discretized by a wavelet transform method to accurately capture the topological change of the fluid-fluid interface. To validate the closure model for interphase forces, the results of the present phase-field method have been compared with that from experiments, as well as from reference numerical models. An excellent agreement among the results from present phase-field simulations, experiments, and some reference numerical simulations has been observed. The terminal velocity of the rising gas bubble in a liquid saturated porous medium, as well as in a pure liquid has been investigated. The bubble rising velocity in both cases have been compared with respect to the theoretical and experimental results. The study illustrates how the bubble dynamics in porous media depend on the excess free energy of a thin interfacial layer formed by two immiscible fluids.

1. Introduction

Numerical modelling of multiphase flow and transport has attracted numerous researchers because of its importance in various scientific and industrial applications (Giorgio et al., 2017; Horgue et al., 2015; Higdon, 2013; Wu et al., 2016). Of particular interests are two-phase flow in and around a wellbore, enhanced oil recovery techniques, and carbon capture and storage projects (e.g. Arzanfudi et al., 2016; Kundu et al., 2016; Rahman et al., 2013). A critical challenge in modelling oil and gas is that the flow rates often cease to remain linear with the pressure gradients in wellbores/reservoirs, and the capillary stress becomes important (Wu et al., 2011; Mahdiyar et al., 2011). To investigate the nonlinear flow behavior in the near wellbore region, Molina and Tyagi

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Nomenclature

x	horizontal coordinate (m)
z	vertical coordinate (m)
t	time (s)
$\boldsymbol{u}$	velocity field (ms^{-1})
P	pressure (Pa)
g	acceleration due to gravity (ms^{-2})
$\mathcal{K}$	permeability of the medium (m^2)
ϕ	porosity of the medium (–)
c_t	gas compressibility (–)
C	volume fraction of fluid (m^3)
I	identity matrix (–)
d	characteristic of pore space (m)
$\mu_{g(l)}$	gas (liquid) viscosity (mPa.S)
ν	kinematic viscosity (m^2/s)
$\rho_{g(l)}$	gas (liquid) density ($\mathrm{kg/m}^3$)
ρ	density ($\mathrm{kg/m}^3$)
P_l	liquid pressure (Pa)
P_g	gas pressure (Pa)
σ	surface tension ($\mathrm{N\,m}^{-1}$)
τ_σ	stress due to surface tension ($\mathrm{N\,m}^{-1}$)
ϵ	interface thickness (m)
κ	curvature of the interface (1/m)
$\mathcal{W}$	total free energy ($\mathrm{N\,m}^{-1}$)
μ^c	chemical potential (J/mole = Newton)
$1/\mathcal{T}$	inverse elastic relaxation time-scale (1/s)
$\Delta x, \Delta y, \Delta z$	grid space along x, y, z-axis
ΔV	grid volume
Δt	time step
O	order of magnitude
μm	micrometer
$\mathcal{N}$	number of grid points

Non-dimensional parameters

Re	Reynolds number
$\mathcal{D}a$	Darcy number
Bo	Bond number
U	velocity scale
D	length scale

Subscripts

i	$i = 1, 2$ phase
g	gas
l	liquid

Abbreviations

CFD	Computational Fluid Dynamics
CPU	Central Processing Unit
EOR	enhanced oil recovery
REV	Representative Elementary Volume
VOF	volume of fluid

(2015) considered the Navier-Stokes equation to simulate the sudden change of shear stress and flow direction in perforation tunnels that are connections between the reservoir and the wellbore (cf. Fig. 1). However, as the reservoir fluids enter into the wellbore, the liquid phase may remain continuous, and the gas phase may appear as randomly distributed bubbles. In such case, the pressure drop may be described by the bubble velocity, friction factor, and buoyancy (Livescu et al., 2010). In contrast, the presence of bubbles in a reservoir would alter some reservoir properties, such as the macroscopic hydraulic conductivity of the medium, and the pressure drop across a bubble may become a nonlinear function of the fluid velocity. Bubbles may plug some pore channels or fractures and reduce the overall flow rate in a reservoir. CFD simulations of such a multiphase flow in reservoirs (or in wellbores) remain elusive primarily because the capillary stress is active over a wide range of length and time scales, and we don't have a widely acceptable closure scheme to truncate the spectrum of the length scales effectively. Technical details of the closure schemes for multiphase flow are covered by Adler and Brenner (1988), Higdon (2013), Alpak et al. (2016).

Consequently, a multiphase bubbly flow in porous media is often studied by the Buckley-Leverett model, or the pressure profile in the reservoir is analyzed by the pressure diffusivity equation. In both approaches, the length scales are truncated based on the

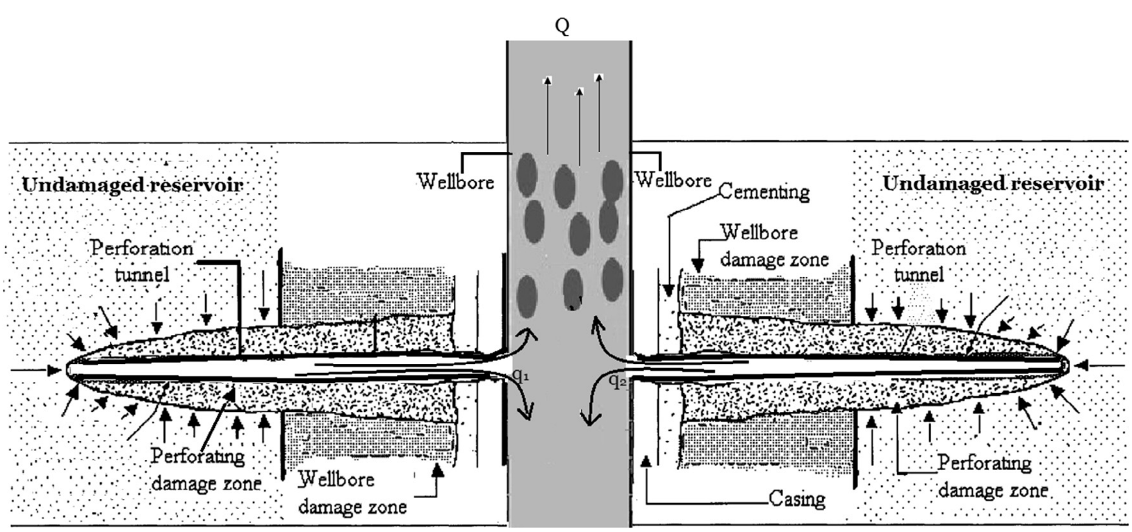

Fig. 1. A schematic illustration of an oil reservoir, where a vertical wellbore with perforation tunnels are also shown. The figure is adapted from a similar illustration given by Rahman et al. (2007). Note that the fluid flow is vertical in the wellbore and horizontal in the reservoir. Bubbles may be present in the wellbore or in the reservoir. When the reservoir fluid enters into the wellbore, the flow rate is affected by the formation damage near the perforation tunnels.