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A complex model for the permeability and porosity of porous media

Xiao-Hua Tan [a,*], Li Jiang [a], Xiao-Ping Li [a,*], Yue-Yang Li [b], Kai Zhang [b]

[a] State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu 610500, China
[b] CNPC Southwest Oil and Gas Field Exploration and Development Research Institute, 610041, China

HIGHLIGHTS

- A complex model for the permeability and porosity of porous media is developed.
- The characters of pore microstructure and fluid in porous media are considered.
- This model reveals the all kinds of fluids flow in variety of porous media.
- The predictions of the model show good agreement with available experimental data.
- The proposed model can be used to characterize the flow in porous media.

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ABSTRACT

Transport phenomenon in porous media is essential in various scientific and engineering fields. The permeability and porosity of porous media serve important functions in transport phenomena. In this study, a complex model that considers the characters of pore microstructure and fluid in porous media is developed for the permeability and porosity of porous media. The flow rate expressions of this model are compared with those of previous models. As the proposed models relate the properties of fluids to the structural parameters of porous media, the various expressions of the developed model can reveal the phenomena of all types of fluid flow in a variety of porous media. The predicted permeabilities and porosities show good agreement with the available experimental data and illustrate that the proposed model can be used to characterize flow in porous media.

1. Introduction

Transport phenomena in porous media have steadily received considerable attention in many fields, such as hydraulics (Escobar et al., 2011; Pia and Sanna, 2014a; Pia et al., 2014), chemical (Cai et al., 2012), petroleum engineering (Dong et al., 2005; Xu et al., 2017). The subject of transport phenomena includes three closely related topics: fluid dynamics (Xiong et al., 2017), heat transfer (Miao et al., 2016), and mass transfer (Yang et al., 2017). Fluid dynamics involves the transport of momentum, heat transfer deals with the transport of energy, and mass transfer is concerned with the transport of mass of various chemical species (Bird, 2002).

Transport phenomena in porous media are still not well-understood. Because of the disordered and extremely complicated microstructures of porous media, it is difficult to describe the transport procession and obtain the transport parameters, such as permeability (Guarracino et al., 2014; Liu et al., 2016; Xu et al., 2016), thermal conductivity (Huai et al., 2007; Pia and Sanna, 2014b), mechanical behavior (Gou and Schwartz, 2013). In this case, fractal theory is introduced to describe the microstructures of porous media and obtain the permeability and porosity of porous media. Because fractures in nature are random and disorder in spaces and lengths, and real porous media have been shown to have the statistically self-similar and fractal characteristic. Tremendous attention has also been directed toward its methods, theories, and progresses in these areas (Xu et al., 2017). The permeability and porosity of porous media serve important functions in transport phenomena, which are usually studied via experiments, numerical simulations, and analyses. Capillary bundle models have been used to predict the permeability of porous media with parameters obtained from experimental data, avoiding the tedious calculations of numerical simulations (Cao et al., 2016; Yang et al., 2014a).

* Corresponding authors.
E-mail addresses: xiaohua-tan@163.com (X.-H. Tan), ncixphm@126.com (X.-P. Li).

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Purcell (1949) assumed that porous media are composed of numerous parallel cylindrical capillaries with equal length but different radii, and that total flow rate through porous media is equivalent to the sum of the contributions made by each capillary. All these assumptions laid the foundation of the capillary bundle model. Purcell took advantage of Poiseuille's equation

$$q = \frac{\pi r^4 \Delta p}{8\mu L}, \tag{1}$$

where q is the flow rate of a single capillary, r is the radius of a capillary, Δp is the pressure gradient along a tortuous capillary, μ is the fluid viscosity, and L is the length of a capillary. The capillary pressure equation is

$$p_c = \frac{2\sigma \cos\theta}{r}, \tag{2}$$

where p_c is the capillary pressure of a single capillary, σ is the interfacial tension, and θ is the angle of contact.

Then, according to the basic assumption of the capillary bundle model, Purcell proposed the flow rate expression of porous media shown as follows:

$$Q = \frac{(\sigma\cos\theta)^2 \Delta p}{2\mu L^2} \sum_{i=1}^{N} \frac{V_i}{p_{ci}^2}, \tag{3}$$

where Q is the flow rate through porous media, V_i is the pore volume of each capillary, and N is the number of capillaries in porous media. The permeability expression of porous media is

$$K = \frac{F\phi(\sigma\cos\theta)^2}{2 \times 10^4} \sum_{i=1}^{N} \frac{S_i}{p_{ci}^2}, \tag{4}$$

where K is the permeability of porous media, F is the pore shape factor, ϕ is the porosity of porous media, and S_i is the saturation of each capillary.

Therefore, Purcell established the basic permeability form of the capillary bundle model, which contains the pore geometrical factor, the porosity of porous media, the saturation, and the capillary pressure. His study was continued by many researchers, as shown in Table 1. The parallel expression was given as

$$K = A\left(\frac{S_b}{p_c}\right)^B, \tag{5}$$

where S_b and p_c are the respective saturation and capillary pressure of porous media. Some researchers (Buiting and Clerke, 2013; Thomeer, 1983, 1960) used saturation at infinite capillary pressure $S_{b\infty}$ and displacement pressure p_d instead of S_b and p_c, but the expression $S_{b\infty}/p_d$ presented the similar relationship between injected fluid saturation and capillary pressure. The parameters A and B, which matched experimental data, presented the pore structure of porous media.

Thomeer (1960) showed the capillary pressure curves based on experiments and proposed the relationship between capillary pressure p_c and saturation S_b, using saturation at infinite capillary pressure $S_{b\infty}$, displacement pressure p_d, and pore shape factor F. He represented the equation for the permeability of porous media as

$$K = A\left(\frac{S_{b\infty}}{p_d}\right)^{2.14}, \tag{6}$$

where A could not be determined because of the limitations of technology at that time.

Twenty years later, Thomeer (1983) found that parameter A is related to pore shape factor and took advantage of experimental data to modify the power-law index from 2.14 to 2. The expression is given as follows:

$$K = 3.8068F^{-1.3334}\left(\frac{S_{b\infty}}{p_d}\right)^2. \tag{7}$$

Swanson (1981) also made an enormous contribution to the permeability expression of porous media. On the basis of the experimental data of 24 clean sandstone samples from 21 formations, Swanson first proposed the correlation as

$$K = 431\left(\frac{S_b}{p_c}\right)^{2.109}. \tag{8}$$

Afterwards, according to 32 carbonate samples representing 13 formations, Swanson gave the second expression as

$$K = 290\left(\frac{S_b}{p_c}\right)^{1.901}. \tag{9}$$

In the end, the overall relationship based on all the experimental data was presented as

$$K = 355\left(\frac{S_b}{p_c}\right)^{2.005}. \tag{10}$$

Wells and Amaefule (1985) proposed a new relationship between the parameters and the permeability for tight gas sands. This relationship led to a more accurate predicted permeability below 10 microdarcies:

$$K = 30.5\left(\frac{S_b}{p_c}\right)^{1.56}. \tag{11}$$

Buiting and Clerke (2013) considered the tortuosity and fractal character of capillary bundles to develop their capillary bundle model. The model can be expressed similar to the formulation of the Swanson permeability expression with new parameters.

$$K = 4316\,e^{-5.67\sqrt{F}}\left(\frac{S_b}{p_c}\right)^{1.87}, \tag{12}$$

where F presents the variability of pore throats in porous media. A large F value means high pore throat variability within the pore system, resulting in a low permeability of porous media.

Kozeny (1927) derived the expression of permeability of porous media by assuming that the porous media comprise a bundle of capillaries with the same diameter. Carman (1937) modified

Table 1
The permeability expressions of Purcell-based models.

Model name	Value of a	Value of b	Dataset description	Reference
Thomeer 1	–	2.14	–	Thomeer (1960)
Thomeer 2	$3.8068F^{-1.3334}$	2	165 siliciclastic and 114 carbonate samples in 54 fields	Thomeer (1983)
Swanson 1	431	2.109	24 sandstone samples in 21 formations	Swanson (1981)
Swanson 2	290	1.901	32 carbonate samples in 13 formations	Swanson (1981)
Swanson 3	355	2.005	24 sandstone and 32 carbonate samples in 21 formations	Swanson (1981)
Wells and Amaefule	30.5	1.56	35 tight gas sandstone samples in 2 fields	Wells and Amaefule (1985)
Buiting and Clerke	$4316\,e^{-5.67\sqrt{F}}$	1.87	More than 500 samples	Buiting and Clerke (2013)