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A dual grid method for the compressible two-fluid model which combines robust flux splitting methodology with high-resolution capturing of incompressible dynamics

A.H. Akselsen

Department of Energy and Process Engineering, Norwegian University of Science and Technology, Kolbjørn Hejes v. 1B, 7491 Trondheim, Norway

HIGHLIGHTS

- A dual grid strategy for improving simulation efficiency is presented.
- Sonic and hydraulic flow information is couple between two distinct numerical grids.
- Prediction accuracy is increased without significant computational expense.
- Simulation efficiency is improved by several orders of magnitude.

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ABSTRACT

The speed of sound in two-phase pipe flow systems is often several orders of magnitude greater than the travelling speed of hydraulic information (volume fractions). Dynamically simulating such flows requires resolution of acoustic and hydraulic waves existing at vastly different spatial and temporal scales. If simulated on the same numerical grid, the need for accuracy in hydraulic waves will necessitate an exaggerated resolution of acoustic waves. Likewise, time steps restricted by the speed of sound are small compared to the time scales active in hydraulic waves. This constitutes a waste of computational potential. The method proposed herein decouples the hydraulic and acoustic scales, greatly improving computational efficiency.

The proposed dual grid method solves a four-equation compressible two-fluid model on a principal grid which robustly accounts for the pressure evolution and conserves mass and momentum. An incompressible two-equation model is at the same time solved on a finer grid, resolving the details of the hydraulic evolution. Information from both model formulations is coupled through the terms of the governing transport equations, providing consistency between the grids. Accurate and finely resolved schemes can then be employed for the incompressible two-fluid model without suffering from the time and stability restrictions otherwise enforced by acoustic waves. At the same time, the Hybrid Central-Upwind flux splitting scheme of Evje and Flåtten (2005a) allows for an explicit and numerically robust treatment of the acoustic waves without losing hydraulic accuracy.

The dual grid method is tested against four dissimilar problems: A shock tube problem, the water faucet problem, a surge wave and pressure wave problem and a roll-wave case. In all problems, the proposed scheme provided significant increases in computational efficiency and accuracy as compared with a single grid arrangement.

1. Introduction

Dynamic flow simulators have been a vital tools in industries like the nuclear industry (Ransom et al., 2001; Barre and Bernard, 1990) and the petroleum industry (Bendlksen et al., 1991; Larsen et al., 1997). Predictions of the flow topology, fluid transport and pressure loss, as well as the simulation of potentially damaging or dangerous scenarios, are among the key features of these simulation tools. Many, if not most, such simulators are based on the so-called two-fluid model, which is derived by averaging the fundamental conservation equations over district flow fields – for example a gas and a liquid field. The mechanism for hydrodynamic growth of long-wavelength instabilities is known to be an inherent

E-mail address: andreas.h.akselsen@ntnu.no

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feature of the two-fluid model (Barnea and Taitel, 1993). Capturing this mechanism has in recent years become a popular method for predicting the flow field topology and flow regime transitions in dynamic simulators (Issa and Kempf, 2003). This methodology has even found its way into commercial software for large scale pipeline systems (Kjolaas et al., 2013; Ransom et al., 2001).

The characteristic speeds active in gas-liquid flows can differ by several orders of magnitude. Sonic waves are artefacts of fluid compressibility and propagate much quicker than hydraulic waves pertaining to changes in the volume fraction. Hydraulic waves are responsible for the hydrodynamic growth of long-amplitude surface waves, often of primary interest. Acoustic or 'sonic' wave (waves in pressure) work on a time scale too small to affect long-amplitude waves significantly, giving the pressure an idle role regarding surface waves. All the same, sonic waves must be computed carefully if the simulation procedure is to remain numerically stable, placing a strict time step restriction on explicit solvers.

A simple way to allow a numerical scheme to operate at the slow time scales suited for hydraulic waves is to ignore compressibility altogether. The four-equation two-fluid model can then be reduced to a two-equation form as two pressure waves are removed from the system. Worth mentioning in regard to this incompressible two-equation model is Keyfitz (2003), who analysed it mathematically. Wangensteen (2010) proposed a flux splitting technique for intermittent single phase – two phase flows (slug flow) built on the two-equation model. He also constructed a Roe scheme based on Keyfitz' formulation. The incompressible two-fluid model has further been used by Holmås (2010) to effectively simulate high-pressure flow in the roll-wave regime. These simulations compared favourably to the experimental campaign of Johnson (2005). Holmås' formulation of the incompressible model was based on the formulation used by Watson (1989), which is somewhat cleaner than the one investigated by Keyfitz. The present author used that model formulation to construct a Roe scheme and schemes based on the principle of characteristics (Akselsen, 2017), proving very efficient.

The present work revisits the dual grid methodology for resolving hydraulic and acoustic waves on separate grids. The concept was investigated in Akselsen and Nydal (2015) using a primitive decoupling that discriminated between gas and liquid phases. The gas phase was associated with acoustic waves and designated to a coarse grid, while the liquid phase was resolved in greater detail. Although the method indicated the potential of the dual grid strategy, it suffered from grid dependent disturbances generated as large scale hydraulic information was projected down onto the smaller scales.

The presently presented method distinguishes between scale based on the compressibility itself, projecting only information pertaining to compressibility down onto the smaller scales. A two-way coupling between the two computational grids is achieved using the flux splitting approach due to Evje and Flåtten (2005a), which also ensures a robust yet explicit treatment of the pressure. At the same time, the presented dual grid scheme exploits the benefits of the incompressible two-fluid model, in particular its neatness and simple eigenstructure. The resulting method is one which successfully neutralizes the difference in sonic and hydraulic travelling speeds. It allows for simple, explicit and affordable simulation in a variety of cases which on a single grid arrangement would require a semi-implicit formulation or significant computer power.

This article is structured as follows: The four-equation two-fluid model for stratified pipe flow is briefly presented in Section 2. Section 3 provides the building blocks of the dual grid scheme. This includes a summary of the Hybrid Central-Upwind (HCU) flux splitting scheme (Section 3.1,) the incompressible two-fluid model with a Roe scheme discretization (Section 3.3,) and the means by which these are coupled (Sections 3.4.1 and 3.4.2.) Linear stability expressions are presented in Section 4.1, while Section 4.2 illustrates how the two-grid arrangement maintains the properties of the HCU scheme presented in Evje and Flåtten (2005a). Numerical tests presented in Section 5 are given in two parts. Two basic benchmark tests from original HCU publication (shock tube and water faucet) are repeated with extra subgrid resolution in Section 5.1. Two larger problems, more closely related to engineering, are studied in Section 5.2. Acoustic-hydraulic wave interactions, computational efficiency and flow regime prediction are considered in these problems. A summary is given in Section 6.

2. The two-fluid model for stratified pipe flows

The compressible, equal pressure four-equation two-fluid model for stratified pipe flow results from an averaging of the conservation equations within a flow field over the pipe cross-section. It is commonly written

$$\partial_t m_k + \partial_x i_k = 0, \tag{2.1a}$$

$$\partial_t i_k + \partial_x (u_k i_k) + a_k \partial_x p + g_y m_k \partial_x h = s_k, \tag{2.1b}$$

$$a_\ell + a_g = \mathcal{A}, \tag{2.1c}$$

$$\rho_k = \rho_k(p). \tag{2.1d}$$

Specific Mass $m_k = \rho_k a_k$ and momentum $i_k = \rho_k a_k u_k$ per unit length are conserved properties. Field k, occupied by either gas, $k = \mathrm{g}$, or liquid, $k = \ell$, is segregated from the other field. p is here the pressure at the interface, assumed the same for each phase as surface tension is neglected. h is the height of the interface from the pipe floor, and the term in which it appears originates from approximating a hydrostatic wall-normal pressure distribution. u_k and ρ_k are the mean fluid velocity and density, respectively, in field k. The momentum sources are $s_k = -\tau_k \sigma_k \pm \tau_\mathrm{i} \sigma_\mathrm{i} - m_k g_x$, where τ_k and σ_k is the skin friction and perimeter of the pipe wall in field k, respectively. τ_i and σ_i refer to the interphase; see Fig. 2.1. $g_x = g \sin\theta$ and $g_y = g \cos\theta$ are the horizontal and vertical components of the gravitational acceleration, respectively. θ is here the pipe inclination, positive above datum.

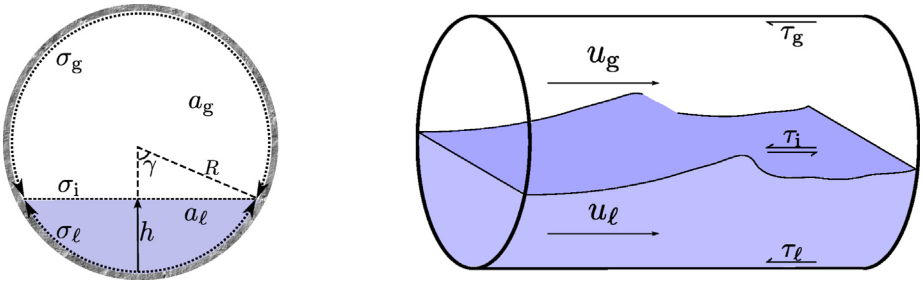

Fig. 2.1. Pipe cross-section.