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Stability of gravity-driven free surface flow of surfactant-laden liquid film flowing down a flexible inclined plane

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HIGHLIGHTS

- Stability of surfactant-laden film flow down a flexible inclined wall is analyzed.
- The wall deformability qualitatively alters the stability behavior of film flow.
- Marangoni mode becomes unstable even in creeping flow due to wall elasticity.

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ABSTRACT

We examined the linear stability of gravity-driven creeping flow of a liquid film flowing down an inclined plane when the inclined plane is coated with a deformable solid layer and the gas-liquid interface is contaminated with a monolayer of insoluble surfactant. The contaminated liquid film flowing down a rigid incline admits gas-liquid (GL) interfacial mode and surfactant-induced Marangoni mode, both of which remain stable in the creeping flow limit. The primary aim of this study is to explore the effect of the presence of a deformable wall, in place of a rigid inclined wall, on the stability behavior of Marangoni mode which originates because of the transport of surfactant at the GL interface. In presence of a deformable solid layer, two additional parameters namely, shear modulus and thickness of deformable solid layer also affects the stability behavior of falling film configuration. Our long-wave asymptotic analysis and results at finite and arbitrary wavenumbers demonstrated the destabilization of Marangoni mode solely due to the presence of deformable solid layer. Specifically, we have shown that for a given solid thickness, the Marangoni mode becomes unstable when the shear modulus of solid layer decreases below a critical value (i.e. the solid layer becomes sufficiently soft). The effect of increasing solid thickness is found to be destabilizing. The LS interfacial mode also becomes unstable at high wavenumbers below a threshold value of shear modulus, however, this value is much smaller than that required to trigger Marangoni mode instability. This implies that as the solid coating becomes more and more deformable, the Marangoni mode becomes unstable first followed by the LS interfacial mode. The GL mode was always found to be stable in creeping flow limit. Further, our long-wave analysis shows that the solid deformability has an additional stabilizing effect on GL mode. The neutral stability curves in nondimensional solid deformability parameter (or equivalently shear modulus) vs. wavenumber plane clearly depicts that the Marangoni mode is the dominant unstable mode in the creeping flow limit. Thus, the present study shows the destabilization of Marangoni mode solely due to presence of deformable solid layer and this we believe is the first example of the case where the instability of the Marangoni mode is observed when the fluid-fluid interface (here, GL interface) remains stress-free in the basic state.

1. Introduction

The stability of gravity-driven liquid film flow with free surface is an extensively studied problem because of its relevance in many engineering applications such as coating processes (Weinstein and Ruschak, 2004), distillation units, condensers, heat exchangers (Craster and Matar, 2009), as well as in biological systems such as pulmonary fluid mechanics (Grotberg and Jensen, 2004; Halpern and Grotberg, 1993). Beginning from the pioneering investigations by Benjamin (1957) and Yih (1963), there have been a large number of studies exploring the onset of instability of a falling liquid film and subsequent non-linear evolution of surface waves at gas-liquid interface. A comprehensive summary of the

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work done in the field is given in Chang and Demekhin (2002) and Craster and Matar (2009). Further, many interfacial flow systems, including the falling film configuration, contain surface active agents or surfactants which play a critical role in different applications (Morrow and Mason, 2001; Quere et al., 1997; Goerke, 1998). The surfactant present at the fluid-fluid interface not only significantly affects the stability behavior of the existing fluid-fluid interfacial mode (Blyth and Pozrikidis, 2004; Samanta, 2014) but also introduces a new additional normal mode originating from the transport of surfactant at the interface (Frenkel and Halpern, 2002). A large number of papers concerning the effect of insoluble or soluble surfactant on the stability of different interfacial flow systems have appeared in the last decade (Halpern and Frenkel, 2003; Blyth and Pozrikidis, 2004; Pozrikidis, 2003; Peng and Zhu, 2010; Gao and Lu, 2007; Samanta, 2014; Bassom et al., 2012; Bassom and Blyth, 2013; Karapetsas and Bontozoglou, 2013; Picardo et al., 2016). However, a majority of the studies considered interfacial flows over a rigid wall. Flow past soft, deformable solid surface occurs in a wide variety of settings; for example, in microfluidic devices made of soft elastomers, flow in lung airways (Halpern and Grotberg, 1992), use of rubber-covered rolls to reduce defects in coating processes (Carvalho and Scriven, 1997), potential use of soft walls in instability suppression (Shankar and Sahu, 2006), inducing ultra-fast mixing and enhancement of mass transfer in microfluidic channels (Bandaru and Kumaran, 2016; Shrivastava et al., 2008). There are several studies which examined the stability and evolution of (clean) liquid film flow down a flexible inclined wall. However, very few studies investigated the stability of surfactant-laden film flow down a flexible incline (Matar and Kumar, 2004; Peng et al., 2016). In this paper, we attempt to fill this gap and examine the linear stability of liquid film falling down an inclined plane when the plane is coated with a soft, deformable solid layer and the free surface is contaminated with a monolayer of insoluble surfactant. In the following, we briefly review the literature and motivate the context of the present study.

Yih (1963) analyzed the linear stability of liquid film flow down an inclined plane for long-wave perturbations and Lin (1967) extended this analysis to finite and arbitrary wavenumbers. It was demonstrated that the liquid film becomes unstable in presence of inertia when the Reynolds number increases above a critical value. This long-wave instability is referred as Yih mode or GL mode or free surface mode in the present work. The dynamics of the falling film is affected by the presence of surface active agents or surfactants. The surface tension depends on the concentration of surfactants and any gradient in surfactant concentration causes a gradient in surface tension which results in the generation of stress known as Marangoni stress. These Marangoni stresses, in turn, could alter the stability behavior of GL interfacial mode. Indeed, earlier works by Whitaker (1964), Whitaker and Jones (1966), Anshus and Acrivos (1967), Lin (1970) all predicted that the presence of insoluble surfactant at free surface has a stabilizing effect on this GL mode and the critical Reynolds number increases for liquid film contaminated with surfactant in comparison to the clean film. The effect of soluble surfactant was also investigated by Ji and Setterwall (1994) and they observed the occurrence of an unstable surfactant mode for a vertical falling film. Blyth and Pozrikidis (2004) presented the numerical solution of Orr-Sommerfeld eigenvalue problem for the gravity-driven flow of a liquid film loaded with insoluble surfactant for both zero and finite Reynolds number. They observed the existence of two normal modes in Stokes flow limit. The first is the usual Yih mode and the second is identified as Marangoni mode which occurs due to the spatial variations in surfactant concentration. Both modes remain stable in zero Reynolds number limit with the decay rate of Marangoni mode being significantly lower than the Yih (or GL) mode. Thus, Marangoni mode is the least stable mode in creeping flow limit (and at low Reynolds number). The Marangoni mode remains unaffected as Reynolds number increases above zero while the growth rate of Yih mode increases with Reynolds number and eventually Yih mode overtakes the Marangoni mode at low Reynolds number. The Yih mode finally becomes unstable beyond a critical Reynolds number and a low-wavenumber asymptotic analysis for Yih mode demonstrated that the presence of surfactant raises the critical Reynolds number for the onset of GL mode instability. The second mode, i.e. the Marangoni mode, was never found to be unstable and hence, the overall effect of surfactant was said to be stabilizing. In a related study, Pereira and Kalliadasis (2008) investigated the same problem in both linear and non-linear regime. The linear regime is examined in the framework of Orr-Sommerfeld formulation using both analytical solution in the limit of low wavenumbers and numerical solution for arbitrary wavenumbers. Their low-wavenumber analysis also revealed the existence of a second normal mode in addition to the usual Yih mode. This second mode originates from the surfactant transport equation and it is this mode which was referred as Marangoni mode in Blyth and Pozrikidis (2004). However, they argued that this additional mode is simply a diffusional mode which would be present even in absence of Marangoni effect- i.e. for any species on GL interface and not necessarily a surfactant. The basis of their argument was that the growth rate was found to be proportional to surfactant diffusivity rather than Marangoni number. This mode will continue to exist even for zero Marangoni number and it is the Marangoni number which characterizes the rate of change of surface tension with respect to concentration. They referred this mode as diffusional/concentration mode and observed that this second mode always remains stable. The characteristics of GL mode was found to be in agreement with the results of Blyth and Pozrikidis (2004). The Marangoni/concentration mode was overlooked in several previous studies in context of film flows possibly because of stable behavior of this mode.

Frenkel and Halpern (2002), Halpern and Frenkel (2003) were the first to discover the presence of a surfactant-induced unstable mode for the case of two-layer channel flow when the fluid–fluid interface is covered with a monolayer of surfactant. They demonstrated that in creeping flow limit, the usual fluid-fluid Yih type interfacial mode remains stable, while, an additional mode (called as Marangoni mode) originating due to the presence of surface tension gradients becomes unstable in Stokes flow limit. They showed that the simultaneous presence of surfactant and basic interfacial shear is sufficient to cause Marangoni mode instability even at zero Reynolds number. They suggested the absence of shear stress at GL interface as the reason for not observing the Marangoni mode instability for falling film configuration. Blyth and Pozrikidis (2004) analyzed the stability of two-layer inclined channel flow in presence of an insoluble surfactant using a lubrication approximation and derived nonlinear evolution equations for interface position and surfactant concentration. Their linear stability results from lubrication flow model and Stoke's flow approximation confirmed the results of Frenkel and Halpern (2002), Halpern and Frenkel (2003) that the presence of interfacial surfactant introduces a Marangoni mode instability for certain values of viscosity and thickness ratio of two fluid layers. Motivated by these observations, Wei analyzed the linear stability of surfactant-laden falling (Wei, 2005 and visco-elastic Wei, 2005) film down an inclined wall with shear imposed on the GL interface. He clearly demonstrated that depending on the direction of imposed shear, the Marangoni mode could be stable or unstable. The effect of imposed shear on GL mode was also discussed. Similar observations were made by Zhou et al. (2014) who investigated the role of imposed shear on the stability of visco-elastic contaminated film falling on the inner surface of a cylindrical tube. These studies re-consolidate the idea