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Interaction of bubbles rising inline in quiescent liquid

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HIGHLIGHTS

- Numerical study on the interaction of two inline bubbles rising in stagnant liquid.
- Trailing bubble interacts with wake of the leading bubble and accelerates beyond terminal velocity.
- At low Reynolds (*Re*) numbers, inline configuration is stable and bubbles collide.
- At higher Re values, trailing bubble deviates from rectilinear path due to vorticity development.
- The onset of path deviation depends on the Reynolds and Eotvos numbers of the bubbles.

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ABSTRACT

In the buoyant rise of two bubbles arranged in an inline configuration, the trailing bubble tends to accelerate beyond its terminal velocity, due to its interaction with the wake of the preceding bubble. It is demonstrated that several different interactive behaviours could be obtained due to this acceleration. Firstly, at low values of Reynolds numbers ($Re = \rho_l V_t D/\mu \leqslant 35$), the inline configuration was found to be stable, and the two bubbles would collide and coalesce due to the velocity difference between the two. Secondly, at higher *Re* values ($Re > 50$), vorticity development around the trailing bubble causes it to deviate away from the inline configuration, thus preventing the occurrence of a head-on collision between the two bubbles. The deviation of the bubble was found occur if a certain critical velocity is exceeded by the trailing bubble. The value of the critical velocity was found to decrease with increasing *Re* values. Further, an increase in Eotvos number ($Eo = \rho_l g D^2/\sigma$) tends to increase the critical velocity, indicating that enhanced bubble deformability actually improves the path stability of the trailing bubble. The interaction of bubbles can therefore significantly influence the tendency of the bubbles to collide and coalesce.

1. Introduction

The interaction of multiple bubbles rising in a liquid is a fundamental aspect in the understanding of bubble column hydrodynamics (Joshi et al., 2017). Despite its apparent simplicity, various aspects remain unknown, such that even the behaviour of two bubbles rising inline with each other – a highly idealised and simplified case in comparison to the complex hydrodynamics of bubble columns – still demands further analysis. The DNS studies of Bunner and Tryggvason (2003), along with Esmaeeli and Tryggvason (2005), on the behaviour of multiple bubbles in an initially quiescent liquid showed that a number of bubble distribution behaviours could be developed over time, depending on the sphericity and deformability of the bubbles. Spherical bubbles rising at Eotvos number ($Eo = \rho_l g D^2/\sigma$) of 1.0 and Reynolds number ($Re = \rho_l V_t D/\mu$) of 35 tend to align horizontally, whereas ellipsoidal (or 'deformable') bubbles ($Eo = 5.0$, $Re = 26.0$) tend to maintain a vertical alignment and form vertical columns/streams. On the other hand, Esmaeeli and Tryggvason (2005) showed that deformable bubbles rising at higher *Re* values ($Eo = 3.0$, $Re = 77.6$) tend to remain uniformly distributed throughout the control volume. This shows that various factors influence the interaction of bubbles, and a further analysis on the contribution of these factors is required for a greater understanding of bubble column dynamics.

The hydrodynamics of rising bubbles can be significantly influenced by the wake characteristics of the neighbouring bubbles. In the case of bubbles with inline configuration, this interaction

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causes a trailing bubble to attain a higher rise velocity in comparison to the leading one. This was experimentally observed by Katz and Meneveau (1996) for small air bubbles rising in distilled water ($D \leq 475$ μm, $Re \leq 35$, $Eo \leq 0.027$). It was observed that the velocity of the trailing bubble continuously increases as the distance between the two decreases, leading to collision and coalescence between the bubbles. Before colliding, the relative velocity ($\Delta V = V_2 - V_1$, where V_1 and V_2 are the instantaneous velocities of the leading and trailing bubbles, respectively) of larger bubbles ($349 \leq D \leq 475$ μm) was found to increase considerably (up to 27% of the terminal velocity), whereas smaller bubbles ($D < 349$ μm) tend to attain a decrease in relative velocity. The difference in these behaviours could be caused by the contribution of pressure gradients in the two systems, which become more dominant at smaller distances.

The experimental observations of Katz and Meneveau (1996) were found to be inconsistent with the numerical results of Yuan and Prosperetti (1994), who conducted a study using a mixed spectral/finite-difference scheme for the interaction of spherical inline bubbles in an axisymmetric domain at $Re \leq 200$. Yuan and Prosperetti (1994) reported the presence of an equilibrium distance (L_e) between two rising inline bubbles as a consequence of the balance between the wake interaction effect and the pressure gradients between the two bubbles. The magnitude of the equilibrium distance relative to the diameter of the bubbles was found to scale with Re. The absence of the equilibrium distance phenomenon in the experimental results of Katz and Meneveau (1996) were attributed towards the non-deformability of the bubbles, which is one of the assumptions of the numerical study of Yuan and Prosperetti (1994), as well as towards the presence of impurities in the liquid. More recently, Watanabe and Sanada (2006) showed that inline bubbles rising at $Eo = 0.00878$ and $30 \leq Re \leq 200$ with an initial distance of $5R$ do tend to rise at a stable equilibrium distance after some time through a series of axisymmetric simulations. However, this behaviour was not replicated in their experiments, conducted in silicone oils with bubbles rising at $0.6 \leq Eo \leq 1.0$ and $5 \leq Re \leq 40$. It was noted that the inline configuration of the bubbles as they approach equilibrium distance is not stable, as some pairs of bubbles tend to escape from the vertical line. Further, it was noted that the equilibrium distance (defined as the bubble distance in the vertical direction just before the bubbles escape from the vertical alignment in cases where it is unstable) tends to be ~3–8 times larger than the predictions of Yuan and Prosperetti (1994). Finally, it was observed that the equilibrium distance is dependent on the initial bubble distance.

Zhang and Fan (2003) have conducted a force balance analysis on the rise of aligned spherical bubbles at $Re \sim O(100)$, considering effects of buoyancy, gravity, drag, added mass and Basset forces. A model was proposed, based on the analytical solution for the far wake region of the leading bubble, to determine the drag force and acceleration of the trailing bubble. Several similar models (Ramírez-Muñoz et al., 2013; Baz-Rodríguez et al., 2014) have since been proposed with some adjustments in the axial velocity profile of the wake region of the leading bubble. Ramírez-Muñoz et al. (2013) suggested the incorporation of an 'artificial origin' parameter to describe the velocity profile. The magnitude of the artificial origin was determined through a numerical analysis of the wake region behind a rising spherical bubble, and was found to be a linear function of Re. It was found that buoyancy, quasi-steady drag and inertial forces are the main factors contributing towards the motion of the trailing bubble in the wake region of the leading one, i.e. effects of history and added-mass forces can be neglected. This analysis was found to apply for cases where the separation distance is greater than $4.0R$. At smaller distances, it was noted that the assumption of one-way interaction in the hydrodynamic of the trailing bubble is no longer valid, and that the balance of pressure gradient with the wake interaction effects on the hydrodynamics of two inline bubbles, as shown by Yuan and Prosperetti (1994), needs to be taken into account. Baz-Rodríguez et al. (2014) incorporated a slightly different fit-equation for the expression of artificial origin, and the updated force model was found to be valid for spherical bubbles rising at $50 \leq Re \leq 300$ and separation distances $\geq 5R$. Furthermore, the hydrodynamic force on the leading bubble was also analysed, incorporating the upward inviscid repulsion due to potential flow generated by the trailing bubble. As a result of this analysis, a prediction of the theoretical equilibrium separation distance, consistent with the findings of Yuan and Prosperetti (1994) and Watanabe and Sanada (2006), were made.

Based on the discussion above, it is clear that various uncertainties still exist regarding the interactive behaviour of the bubbles, particularly regarding the presence of equilibrium distance, the stability of the alignment of the bubbles, and the rise velocity of the bubbles as they get closer to each other, i.e. in the near-wake region of the leading bubble. Further, as most air–water bubbly flows exhibit bubbles of radius ≈ 1.5 mm, typical Re and Eo values in practical industrial situations are ~880 and 1.2, respectively (Duineveld, 1998). On the other hand, the studies so far consider the interaction of inline spherical bubbles at low values of Eo, albeit at relatively high Re values ($Re \leq 300$). In this study, therefore, the interaction of two inline bubbles at various Re values at $0.5 \leq Eo \leq 1.0$ will be analysed through 3D numerical analysis. The methodology used in this study will be discussed in Section 2, whereas the parameters of the study will be outlined in Section 3, followed by a discussion on the interactive behaviour of the bubbles.

2. Numerical methodology

The Gerris flow solver (Popinet, 2003) was used for the computation of conservation equations for incompressible flows with variable density and surface tension,

$$\nabla \cdot u = 0, \tag{1}$$

$$\rho(\partial u/\partial t + u \cdot \nabla u) = -\nabla p + \nabla \cdot (2\mu D) + \sigma \kappa \delta_s n. \tag{2}$$

u is the velocity vector, D is the deformation tensor, σ the surface tension coefficient, κ the curvature of the interface, δ_s is the Dirac function, and n is the normal vector to the interface. All other variables (pressure, density and viscosity) have been assigned with their conventional notation.

The basis of the Volume of Fluid (VOF) methodology is the assignment of a scalar variable, α, to indicate the volume fraction of the primary phase in a computational cell. An α value of 1.0 indicates that the cell is filled with the primary phase, and the opposite is true for an α value of 0. The volume fraction field is advected with the local velocity field,

$$\partial \alpha/\partial t + \nabla \cdot (\alpha u) = 0. \tag{3}$$

The density and viscosity of the fluid are calculated at each computational cell based on the value of α, and the densities and viscosities of the individual phase,

$$\rho = \alpha \rho_1 + (1 - \alpha)\rho_2, \tag{4}$$

$$\mu = \alpha \mu_1 + (1 - \alpha)\mu_2. \tag{5}$$

The solver employs staggered temporal discretisation for the solutions of Eqs. (1)(3), which results in a scheme that is second-order accurate in time (Popinet, 2003). Furthermore, hierarchical octree is used to spatially discretise the 3D domain, such that adaptive mesh refinement can be implemented with minimal impact to accuracy. To minimise problems with parasitic currents in the