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Stability analysis by means of information entropy: Assessment of a novel method against natural circulation experimental data

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HIGHLIGHTS

- The Information Entropy (IE) can be adopted to study the dynamic behaviour of a system.
- Transients reaching a steady state have a smaller IE compared to the oscillating ones.
- The IE of a sinusoidal signal represents a well-defined convergence/stability threshold.
- IE can be adopted to compute the stability map for a given system.
- The proposed method has been assessed against natural circulation experimental data.

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ABSTRACT

In this paper, a method based on the Information Entropy (IE) is developed to evaluate the equilibrium stability of a given dynamic system. While for analytical/semi-analytical approaches the definition of stability is formally rigorous (e.g., thanks to the tools provided by the linear analysis), the process of identifying stable and unstable behaviours can be subject to a certain degree of arbitrariness in case of experimental and/or numerical transients. Generally speaking, the classification is based on the time dependent behaviour of the signals recorded during a transient of the system. These signals can be characterised by oscillations with non-decreasing amplitude or can converge to a steady-state value. In the first case, the system experiences an unstable operating condition, in the latter one, the operating condition is stable. For this reason, the key issue is the determination of a well-defined threshold in order to separate converging and oscillating signals. To this purpose, the proposed method evaluates the convergence of a transient by computing the IE associated with a selected signal, and adopts as convergence threshold the IE related to a constant amplitude sinusoid, which represents the condition for the onset of the instability.

In this work, the developed methodology, which can be applied in general to any kind of signal, is assessed against the data obtained from the L2 single-phase Natural Circulation Loop (NCL) (University of Genoa, DIME-Tec Labs), for which the IE is also adopted to evaluate the system stability map. Satisfactory results are achieved not only for the identification of stable and unstable transients, but also for the stability map, which is in agreement with the predictions achievable with other methodologies (e.g., semi-analytical linear analysis) developed by the authors (Pini et al., 2016; Cammi et al., 2016a; Luzzi et al., 2017).

1. Introduction

From a theoretical point of view (Lyapunov, 1892), the stability of an equilibrium state can be analysed by applying a small perturbation (if compared with the reference value) to the system under examination. If the system spontaneously returns to the original state after a transient of infinite length, the equilibrium of the system is asymptotically stable. Otherwise, if the perturbation brings the system in a different equilibrium state or if an equilibrium state is never reached, the original equilibrium is unstable.

The adoption of this simple and elegant definition of asymptotic stability, which is useful in case of analytical or semi-analytical models, is not trivial in experimental and numerical tests. First of

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Nomenclature

Latin symbols

a	uniform distribution lower bound (–)
b	uniform distribution higher bound (–)
$\mathcal{B}$	binomial probability distribution (–)
D	diameter (m)
g	gravity acceleration (m s^{-2})
Gr_m	modified Grashof number (–)
h	information content (nat)
H	height of the L2 facility (m)
$\mathcal{H}$	information entropy (nat)
i	generic index (–)
l	autocorrelation delay (a.u.)
l'	number of points neglected in the end-part of the signal for the definition of RR' (–)
L	length (m)
m	signal sampling index (–)
M	signal sampling number (–)
MM	maximum autocorrelation delay (–)
n	binomial trials (–)
N	total number of possible outcomes (–)
O_X	random process (–)
p	probability (–)
Pr	Prandtl number (–)
q	success probability (–)
q''	localized heat flux (W m^{-2})
Re	Reynolds number (–)
RR	autocorrelation function (–)
RR'	relative square deviation function (–)
$\widetilde{RR}$	windowed autocorrelation (–)
$\widetilde{RR}'$	windowed relative square deviation function (–)
t	time (s)
T	temperature (K)
$\mathcal{U}$	uniform probability distribution (–)
W	width of the L2 facility (m)
x	generic outcome of a random process (a.u.)
X	random variable (a.u.)
y	single value of a signal (a.u.)
$\langle y \rangle$	average value of a signal (a.u.)
Y	generic signal (a.u.)

Greek symbols

σ^2	variance (a.u.)
ψ	Hanning window function (–)

Subscripts-superscripts

b	logarithm base
c	cooler
h	heater
i, j, k	generic index
in	inner
out	outer
t	total length of the loop

Acronyms

1D	one dimensional
a.u.	arbitrary units
amb	ambient
DYNASTY	DYnamics of NAtural circulation for molten SalT internallY heated
ESM	Entropic Stability Map
FEM	Finite Element Method
FFT	Fast Fourier Transform
IE	Information Entropy
IED	Information Entropy Difference
IESA	Information Entropy Stability Analysis
NCL	Natural Circulation Loop
O-O	Object-Oriented
PDS	Power Density Spectrum
RSD	Relative Square Deviation
T1,...,T30	thermocouple No. 1,...,thermocouple No. 30

all, the definition of stability is usually referred to the asymptotic behaviour and thus to infinite-length transients. In addition, while for linear systems the evolution of the state variables after the perturbation follows a sinusoid modulated in magnitude by a decreasing (increasing) exponential for stable (unstable) equilibria, in case of nonlinear systems, more complex behaviours are possible (e.g., limit cycles) and hence the identification of a stable/unstable system equilibrium is more difficult.

An interesting example of systems characterised by a complex dynamic behaviour is given by Natural Circulation Loops (NCLs). NCLs are usually vertical rectangular or toroidal circuits, in which the action of the buoyancy force induces the motion of the working fluid between a hot source and a cold sink. In literature, NCLs are the subject of several works. The first studies were carried out by Keller (1966) and Welander (1967), and more recently by Chen (1985), Vijayan (2002), Vijayan et al. (1995, 2007), Ambrosini and Ferreri (1998, 2000), Ambrosini et al. (2004), Misale et al. (1999, 2005), Misale and Garibaldi (2010), Pilkhwal et al. (2007), Swapnalee and Vijayan (2011), Luzzi et al. (2017), and Srivastava et al. (2016). In NCLs, the equilibrium state is achieved when the driving buoyancy force is in balance with the frictional one. Such equilibrium can be either dynamically stable or unstable.

In this regard, the classification of the system stability on the basis of experimental or numerical data collected into signals is usually carried out by directly observing the time-evolution of some fluid variables such as temperature, pressure and/or mass flow rate (e.g., Ambrosini et al., 2004; Misale et al., 2005; Pilkhwal et al., 2007; IAEA, 2014). If such quantities converge to a steady-state value, the operating condition is considered stable. On the contrary, if the working fluid shows a pulsating behaviour characterised by a non-decreasing amplitude of the oscillations, the operating condition is classified as unstable. Another strategy (e.g., Pini et al., 2016; Cammi et al., 2016a) is based on the computation of ratio between the variance of the final part of the signal and that one of the whole signal. These approaches are intuitive, widely used, and give satisfactory results, but share the limit of introducing arbitrary parameters (e.g., amplitude and maximum variance of oscillations) to define the system stability threshold. In order to overcome this limit, the present paper illustrates a method based on the Information Entropy (IE).

The IE was formulated for the first time by Shannon (1948) to evaluate the information content of the values of a stochastic variable. IE is used in several works to process signals (e.g., see Bollt and Skufca, 2009; Yi-bing et al., 2012; Phung et al., 2014). Moreover, it is widely adopted in informatics for data compression (e.g., Coifman and Wickerhauser, 1992), for cryptography (e.g., Skòrski, 2015), and to evaluate the residual errors during numerical simulations (Brown, 2006).

The IE is also employed for the quantification of the information contained in signals (e.g., the pressure ones) recorded from multiphase thermal-hydraulic systems to study the flow behaviour. In this regard, several methods can be successfully adopted to extract