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# Stability and convergence of computational eulerian two-fluid model for a bubble plume

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ABSTRACT

The Eulerian two-fluid model (TFM) of Ishii (1975) is used to analyze the dynamics of an air-water bubble plume. The focus is on the effect of the linear stability, in particular the ill-posed condition, on the nonlinear stability of the TFM. It is well-known that the TFM for bubbly flows is ill-posed as an initial value problem in the absence of short wavelength physics for non-zero slip velocities. It is also known that the 1-D TFM can be made conditionally well-posed (for void fraction < 26%) by adding momentum transfer due to interfacial pressure difference and virtual mass. However, there is still the possibility of a TFM being ill-posed in regions of higher void fractions (void fraction > 26%). Physically, as the void fraction increases, the bubbles tend to undergo collisions, and the momentum transfer due to this mechanism may become significant. In the current study a bubble collision model adapted from the work of Alajbegovic et al. (1999) is used for CFD TFM calculations for bubbly flows using an LES approach. It is shown by linear stability analysis and non-linear simulations that the collision term makes the TFM unconditionally well-posed and stable in a non-linear sense.

Secondly, computational grid convergence tests are performed with the well-posed CFD TFM. It is observed that the coarse grid solution exhibits an unphysical limit cycle behavior that is inconsistent with turbulence, but as the mesh is refined the solution becomes chaotic. CFD TFM simulations commonly employ a grid restriction to avoid ill-posed behavior. However, this is unnecessary with a well-posed Eulerian TFM derived from first principles using the continuum assumption. Once the restriction is removed by adding appropriate short-wavelength physics, i.e., interfacial pressure difference and collision mechanism, convergence may be approached in a statistical sense consistent with a turbulent CFD model.

## 1. Introduction

The current research is aimed at formulating a nonlinearly stable TFM, i.e., a bounded model, applicable for LES computations of vertical bubbly flows. The paper focuses on the impact of the linear stability on the nonlinear stability and convergence of a CFD TFM for a bubble plume.

In general, the Eulerian TFM has been derived from first principles using three different averaging approaches by Ishii (1975), Drew and Passman (1998), and Vernier and Delhaye (1968). It consists of a set of continuity, momentum and energy equations for each of the constituent phases. The velocity fields are treated separately, which makes the TFM an attractive option to analyze transient phenomena. It has been concluded by Ishii and Hibiki (2011) that the TFM may become very elaborate and all the numerical TFMs are incomplete, rigorously speaking. It is not unusual to neglect complicated terms in the model out of expediency. However, one always risks neglecting terms that may be essential to model the dominant physics and the proper stability. This may render the model incomplete leading to the linear ill-posed condition, which is a common behavior of the TFM. The prevalent practice is to use numerical viscosity or artificial physics to regularize an ill-posed model in order to avoid unphysical short wave instabilities. However two-phase flows are characterized by a number of physical instabilities that arise due to the difference in densities or velocities between the two phases such as Rayleigh-Taylor, Kelvin-Helmholtz, jet and plume instabilities, and it is important that the numerical TFM be capable to reproduce them. When excess regularization is used, the TFM eliminates the physical instabilities along with the unwanted numerical instabilities, and hence it loses some ability to model the flow dynamics. Besides, the use of numerical regularization tends to make the problem grid dependent. The approach followed here is to introduce appropriate short wave physics in order to make the TFM well-posed and convergent while preserving the typical unstable but bounded behavior of a vertical bubble plume.

Linear stability analysis is often used to determine the well-posedness of the TFM formulated as an initial value problem. The

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**Nomenclature**

*Latin*

| | |
|---|---|
| $A$ | Cross-sectional area ($m^2$) |
| $D_b$ | bubble diameter (m) |
| $Eo$ | Eotvos number |
| $g$ | acceleration due to gravity ($ms^{-2}$) |
| $j$ | volumetric flux ($ms^{-1}$) |
| $k$ | wave number ($m^{-1}$) |
| $M$ | Interfacial momentum transfer ($Nm^{-3}$) |
| $p$ | pressure ($Nm^{-2}$) |
| $r$ | density ratio |
| $Re_p$ | particle Reynolds number |
| $u$ | velocity ($ms^{-1}$) |

*Greek*

| | |
|---|---|
| $\alpha$ | volume fraction |
| $\Delta$ | filter size (m) |
| $\Delta x$ | grid size (m) |
| $\lambda$ | eigenvalue ($ms^{-1}$) |
| $\lambda_k$ | wavelength (m) |
| $\nu$ | kinematic viscosity ($m^2s^{-1}$) |
| $\rho$ | density ($kgm^{-3}$) |
| $\sigma$ | surface tension ($N\,m^{-1}$) |
| $\tau$ | shear stress ($kg\,m^{-1}s^{-2}$) |
| $\chi$ | radial distribution function |
| $\omega$ | growth rate ($s^{-1}$) |

*Subscripts*

| | |
|---|---|
| 1,l | liquid phase |
| 2,$g$,$b$ | vapour phase |
| $Coll$ | collision |
| $D$ | drag |
| $L$ | lift |
| $i$ | interfacial |
| $p$ | interfacial pressure |
| $r$ | relative |
| $VM$ | virtual mass |

work of Stuhmiller (1977) is the first published work on the mathematical analysis of the TFM applied to vertical bubbly flows. The inclusion of the momentum transfer term due to interfacial pressure difference was seen to render real eigenvalues and make the TFM well-posed. Lyczkowski et al. (1978) observed complex-valued characteristics for the Eulerian TFM and demonstrated that adding appropriate differential terms in the governing equations eliminated ill-posedness. Pauchon and Banerjee (1986) studied the eigenvalues for the TFM applied to vertical bubbly flows, and observed that by adding momentum transfer due to virtual mass in addition to the interfacial pressure difference, the TFM formulation becomes well-posed as an initial value problem for void fraction, $\alpha_2 < 26\%$. It was concluded (Pauchon and Banerjee, 1986) that the transition of eigenvalues associated with the material waves from being real-valued to complex-valued may be attributed to the flow regime transition. However, there may be situations where the local $\alpha_2$ may exceed 26%, when the flow still consists of finely dispersed bubbles and the TFM is expected to be well-posed. Research work including Park et al. (1998), Drew and Passman (1998), Vaidheeswaran and Lopez de Bertodano (2016), and Vaidheeswaran et al. (2016) analyzed the effect of the shape dependent interfacial pressure and virtual mass coefficients on the TFM which remains conditionally well-posed. The objective of the current analysis is to make the TFM applied for vertical bubbly flows well-posed over the entire range of void fraction seen in practical applications. It is known that the bubbles undergo significant interactions with each other as the void fraction increases. Hence, the effect of adding momentum transfer due to collisions is analyzed using linear and non-linear stability analyses. The effect of collisions is very rarely incorporated in bubbly flow TFMs but it is common practice in particle flow models derived from kinetic theory where it appears as the "particle pressure". It is seen that the TFM becomes unconditionally well-posed when the bubble-bubble interactions are considered using the collision model of Alajbegovic et al. (1999). Nigmatulin et al. (1996) have briefly discussed the impact of such an hydrodynamic interaction term on the well-posedness of the two-fluid model, however the linear and the non-linear stability analyses have not been performed. It is important to note that the analysis of two-fluid model formulated as multi-dimensional initial boundary value problem is very rigorous, and it is difficult to determine its well-posedness as concluded by Enwald et al. (1996). The focus here is to analyze the one-dimensional equations to obtain a well-posed TFM.

In the present work, an attempt is made to extend the findings of 1-D linear stability analyses to 3-D CFD TFM calculations. The bubble plume experiments of Vanga (2004) are considered for the current analysis. A considerable amount of work has been done in the past to model transient bubbly flows using Euler-Lagrange and Euler-Euler approaches. Both the approaches are discussed in detail in Sokolichin et al. (1997), Kuipers and van Swaaij (1997), and Jakobsen et al. (1997). Further, it was shown by Sokolichin et al. (1997) that the two methods produce identical instantaneous flow structures and time averaged properties when a total variation diminishing (TVD) based higher order discretization is used. Given the computational efficiency for large scale domains, the Euler-Euler approach is most commonly used. Pfelger et al. (1999) found that the CFD calculations with the $k$-$\varepsilon$ model are capable of predicting two-phase flow parameters reasonably well. Deen et al. (2001) observed that an LES approach is required to capture the transient behavior of the plume. The LES approach to model bubbly flows also include the works of Lakehal et al. (2002), Milelli (2002), Zhang et al. (2006), Dhotre et al. (2008), Niceno et al. (2008), Ashraf Ali and Pushpavanam (2011), Tomiyama et al. (2002), Ma et al. (2016). The work reported in the literature so far have not considered the interfacial pressure difference or a collision mechanism. In the current study, the Eulerian TFM with these mechanisms is implemented in Ansys Fluent 15.0 to analyze the bubble plume dynamics. Turbulence modeling is achieved using a Large Eddy Simulation (LES) approach. It is demonstrated that irrespective of the dimensionality of the problem, ill-posedness of the basic TFM persists. Implementing momentum transfer due to interfacial pressure difference and collision resolves this issue, thus enabling a grid convergence verification.

It is worth mentioning that the other multi-dimensional aspects of the TFM are also relevant to analyze bubble plumes. The transverse lift force with a positive lift coefficient (applicable for smaller bubbles) stabilizes the TFM as demonstrated by Lucas et al. (2005). A detailed modeling of turbulence related effects on the interfacial momentum transfer would include factors such as non-rectilinear motion of the bubbles, path instabilities, and shape oscillations as highlighted in Saffman (1956), Mercier et al. (1973), Brucker (1999). Diffusion terms related to these mechanisms are considered to have a stabilizing effect on the TFM equations (Pauchon and Banerjee, 1988; Vreman, 2011). However, accurate modeling of these diffusion effects in the Eulerian TFM framework is a challenging task, and beyond the scope of the current study.

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