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Trajectory analysis of a liquid filament under Rayleigh breakup conditions created from laminar rotary spraying of oil-in-water emulsions

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HIGHLIGHTS

- High-speed video data from a rotary sprayer's laminar liquid filament were analyzed.
- Filament Plateau-Rayleigh instabilities were traced across high-speed video frames.
- Fluid velocity and dominant forces were determined from tracking instabilities.
- The liquid filament arc-shape is described using parametric equations.
- The model was validated against image data and was in good agreement.

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ABSTRACT

A liquid jet resulting from the laminar rotary spraying of an oil-in-water emulsion (O/W) with a zero shear viscosity of 60 mPa s and a surface tension on the order 40 mN/m has been studied by means of a high-speed camera. The liquid flow rate and rotational speed of the rotary sprayer are tuned so that the liquid filament breaks up into droplets under Rayleigh breakup conditions. From the high-speed imagery we consider in detail the dominant forces, which define the shape of the liquid jet near the nozzle exit. We use the formation of Rayleigh disturbances as a tracing mechanism across multiple high-speed video frames to determine the role that rotational forces, surface tension, viscous forces and wind resistance play on the shape of the liquid filament as well as the formation of resulting droplets. From this analysis it is determined that rotational forces play the dominant role, thus resulting in a simplified parametric model of the liquid jet trajectory based on the Rossby number only. This model is compared to a previous model defined in the Frenet-Serret frame of reference and shown to be the same under our simplifying assumptions. Image analysis of the liquid filament trajectories shows that the assumptions made in the model are valid under the experimental conditions considered here.

1. Introduction

The difficulty of spray processing emulsions arises from their sensitivity to fluid mechanical stress, which under standard spray conditions causes the emulsive structure to break down. This in turn causes loss of encapsulation of functional components that drive the need for the emulsive structures in the first place (Munoz-Ibanez et al., 2015; Dubey and Windhab, 2013; Dubey et al., 2011; Soottitantawat et al., 2005; Jafari et al., 2008; Rodriduez-Huezo et al., 2004).

Rotary spraying of liquids under laminar conditions is one method for handling shear sensitive materials. This spray regime is a jetting regime characterized by low Reynolds and Weber numbers, in which the liquid jet breaks up due to Plateau-Rayleigh instability (Reitz and Bracco, 1986; Ohnesorge, 1936) or what we will refer to as Rayleigh breakup, that is liquid filament breakup due primarily to the effect of surface tension. By maintaining low Reynolds and Weber numbers, the shear stress remains low, thus reducing loss of encapsulation due to nozzle and spraying effects (Dubey, 2013). Furthermore, the coupling of rotary spraying with Rayleigh breakup of the liquid jet produces droplets having a narrow drop-size distribution with the added benefit that the filament stretching causes filament diameter to reduce, therefore reducing droplet size (Walzel, 2010; Schröder and Walzel, 1998).

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Measurements of Rayleigh breakup in rotational systems were taken by Kitamura et al. (1977) using a stroboscope and camera. Measurements carried out by Wong et al. (2004) used a high-speed camera. They characterized experiment conditions using the dimensionless Weber (We), Reynolds (Re) and Rossby (Rb) numbers as follows: $0.5 < \mathrm{We} < 25$, $1 < \mathrm{Re} < 1000$ and $0.2 < \mathrm{Rb} < 4.0$. Partridge et al. (2005) measured Rayleigh breakup on a pilot scale rotary sprayer with $1 < \mathrm{We} < 275$, $1 < \mathrm{Re} < 1000$ and $0.13 < \mathrm{Rb} < 4.4$. Gramlich and Piesche (2012) used a rotating cup sprayer to study Rayleigh breakup behavior with $5 < \mathrm{We} < 100$ and $0.001 < \mathrm{Ro} < 10$, where Ro is the rotational number.

A common approach for analyzing the Rayleigh breakup behavior of a liquid filament into droplets is to approximate the liquid filament as an axisymmetric jet, since the curvature of the filament arc is large in comparison to the radius of the filament (Ashgriz and Yarin, 2011). The axisymmetric jet is then analyzed according to Rayleigh breakup theory (Rayleigh, 1879; Weber, 1931), which has been discussed rigourously by Eggers and Villermaux (2008), Chandrasekhar (2013) and Lefebvre (1989) as well as others. This approach has been employed by a number of authors, including Gramlich and Piesche (2012), Decent et al. (2009, 2002), Wallwork et al. (2002) and Părău et al. (2007) to analyze Rayleigh breakup behavior of liquid filaments formed by rotary spraying under laminar conditions.

Approximating the liquid filament as an axisymmetric jet requires a set of equations defining the trajectory of the liquid filament coupled with conservation of mass and momentum. In particular, the fluid velocity, pressure and filament radius are defined as a function of material properties, boundary conditions and position. In the analysis of Gramlich and Piesche (2012), Mescher (2012) and Părău et al. (2007), the formulation is set up in a Lagrangian framework, which after simplifying results in a system of equations in a Frenet-Serret frame of reference containing a tangential component, a principle normal component and a binormal component, the last of which may be zero.

This article addresses only the arc-shape and trajectory of the liquid filament, which is jetting from rotary sprayer under laminar conditions. Rather than coupling the conservation equations directly into the model of the fluid trajectory, a force balance is applied to an arbitrary fluid element in a rotating frame of reference. The resulting equations of motion, when translated to an inertial frame of reference, result in a Cartesian formulation of position and velocity for all points along the trajectory.

The goal of this paper is to establish the performance of this simplified model under experimental conditions. The model was verified using high-speed video capture of an oil-in-water emulsion liquid filament jetting from a laminar rotary sprayer. Image analysis of the liquid filament trajectories shows that the assumptions made in the model are valid under the experimental conditions considered here. Moreover, the arc-shape of the trajectories compares well with those predicted by the model.

2. Mathematical model

A liquid filament with an initial nozzle speed w_0 exits the nozzle of a rotary sprayer with radius of the rotor r_0 and constant angular speed ω as shown in Fig. 1. The shape of the filament curve is determined primarily by a force balance among inertial forces, viscous forces, surface tension, gravity, and wind resistance.

In analyzing the shape of the curve formed by the filament, we assume that the liquid filament behaves as a chain of minuscule liquid segments, where intra-particle forces within the liquid segments do not significantly affect the shape of the curve. In this chain adjacent fluid segments are coupled together by viscous and surface tension forces, which act in the axial direction of the fluid segments. Thus, viscous and surface tension forces are grouped together as the tension vector, denoted by $\mathcal{T}$, which by convention points in the negative tangential direction of the liquid filament.

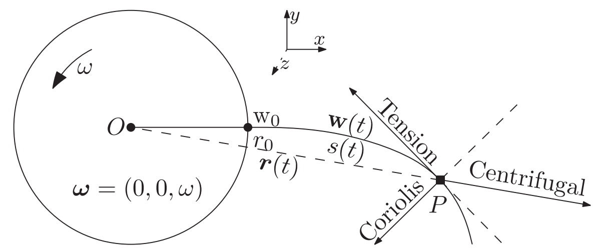

Fig. 1. Force balance diagram of the liquid filament in a rotating frame of reference.

The analysis is performed in a rotating frame of reference with origin O. In the rotating frame of reference centrifugal force and Coriolis force accelerate each filament segment and the remaining force balance remains unchanged. The resulting trajectory of the filament segment is defined as a function of time, t, by the position vector $\mathbf{r}(t)$ with arc length $s(t)$ and velocity vector $\mathbf{w}(t)$.

We assume that inertial forces are significantly greater than the force due to gravity, that is, $w_0^2/r_0 \gg g$, where g denotes the acceleration due to gravity. Therefore, the remaining force balance and fluid motion are restricted to the xy-plane. The position vector then becomes $\mathbf{r}(t) = (x(t), y(t), 0)^{\mathsf{T}}$ and the velocity vector becomes $\mathbf{w}(t) = (u(t), v(t), 0)^{\mathsf{T}}$. Likewise, we assume that wind resistance plays a small role in determining the trajectory. Then the force balance at an arbitrary point, P, on the trajectory determines the acceleration $\mathbf{w}'(t)$ of a segment with length δs and per unit length density, ρ, as follows:

$$\rho\delta s\mathbf{w}'(t) = -\rho\delta s\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}(t)) - 2\rho\delta s\boldsymbol{\omega}\times\mathbf{w}(t) + \mathcal{T}. \quad (1)$$

The magnitude of the tension depends on the position along the trajectory, which is implicitly defined in terms of time. Furthermore, the tension force vector is parallel with but acting opposite to the velocity. Therefore rewriting Eq. (1) in component form, dividing by segment mass, and writing in dimensionless terms results in the parametric form as follows:

$$\ddot{X}(T) = \frac{X(T)}{\mathrm{Rb}^2} + \frac{2}{\mathrm{Rb}}\dot{Y}(T) - F(T)\dot{X}(T)$$
$$\ddot{Y}(T) = \frac{Y(T)}{\mathrm{Rb}^2} - \frac{2}{\mathrm{Rb}}\dot{X}(T) - F(T)\dot{Y}(T), \quad (2)$$

where $F(T)$ represents a scaling of the tension forces and the overhead dot represents the derivative $\frac{d}{dT}$. The equation uses the scaling:

$$X = \frac{x}{r_0}, \quad Y = \frac{y}{r_0}, \quad T = t\frac{w_0}{r_0}, \quad \mathrm{Rb} = \frac{w_0}{r_0\omega},$$

where Rb is the Rossby characteristic number, which represents the ratio of inertial forces to Coriolis force. The dimensionless terms for the arc length, position vector and velocity vector follow from the scaling, which results in:

$$S(T) = \frac{s(t)}{r_0}$$
$$\mathbf{R}(T) = \frac{\mathbf{r}(t)}{r_0} = (X(T), Y(T))^{\mathsf{T}}$$
$$\mathbf{W}(T) = \frac{\mathbf{w}(t)}{w_0} = (U(T), V(T))^{\mathsf{T}},$$

where $U(T)$ and $V(T)$ are the dimensionless velocity components in the x-direction and y-direction, respectively. Since the fluid motion is nonzero only in the xy-plane, the zero-valued z-component of all vectors has been omitted in the remainder of the paper for notational convenience.