Chemical Engineering Science 158 (2017) 1–20

Contents lists available at ScienceDirect

Chemical Engineering Science

journal homepage: www.elsevier.com/locate/ces

Frequency domain constrained optimization of boundary control action for maximization of mixing in channel flow

Pesila Ratnayake, Jie Bao *

School of Chemical Engineering, The University of New South Wales, UNSW, Sydney, NSW 2052, Australia

HIGHLIGHTS

- Developed a constrained optimization approach for maximization of mixing due to electro-osmosis.
- Dynamic voltage inputs (amplitude, phase and frequency) that maximize mixing are determined.
- Developed an infinite-dimensional system to describe the relationship between hydrodynamics and electro-osmotic slip velocities.
- Constructed a virtual output that allows for hydrodynamic mixing maximization to be conducted in the frequency domain.
- The effects of Reynolds number, electrode size, spacing, and number of electrodes were studied.

ARTICLE INFO

Article history:
Received 11 November 2015
Received in revised form
28 June 2016
Accepted 9 September 2016
Available online 20 September 2016

Keywords:
Constrained optimization
Distributed parameter systems
Hydrodynamic mixing
Infinite-dimensional systems
Electro-osmosis

ABSTRACT

Improving mixing is one of the important goals in flow control, *e.g.*, to decrease concentration polarization in membrane systems to reduce fouling. As with many distributed parameter systems, fluid flow can be controlled using boundary value manipulation. Fluid manipulation using electro-osmosis is studied in this work, where several cylindrical electrodes are used to create multiple spatially non-uniform time-varying electric fields. The proposed approach converts the distributed parameter system into an infinite-dimensional system by spatial and spectral discretization. A virtual output variable is constructed to allow the optimization of a mixing objective function to be conducted using frequency response analysis, with consideration of the constraints of conservation of charge. The solution obtained in this paper is the input profile that provides the greatest achievable ratio of time-average dissipation function to time-average input energy satisfying the input constraints.

1. Introduction

Mixing in fluid systems is of great value in applications such as pharmaceuticals and biotechnology (Couchman and Kerrigan, 2010). The aim of mixing in fluids is typically to combine fluids of different properties and make the fluid more homogeneous in terms of properties such as temperature and solute concentrations (Couchman and Kerrigan, 2010). Homogeneous solutions are generally more desirable and products made from homogeneous solutions are generally more consistent and more profitable than those made from poorly mixed fluids (Couchman and Kerrigan, 2010). Laminar flow systems are of particular interest for mixing enhancement. This is because, under laminar flow conditions, very little mixing occurs naturally. Turbulent flows tend to have higher mixing than laminar flows, but this is often accompanied by large energy costs. Additionally, turbulent flows cannot be used in some applications. These include microchannels where the channel size restricts flow to laminar conditions, food processing where the fluids typically have a high viscosity, pharmaceuticals where throughput is generally small, and biotechnology where the materials in the flow may be damaged in high shear conditions (Couchman and Kerrigan, 2010).

In order to mathematically study mixing and develop mixing enhancement approaches, many existing approaches use a scalar function that represents the inhomogeneity of the fluid. Other approaches include using mixing indices such as the fluid stretching, folding, vorticity, and dissipation as scalar functions of interest (Ouyang et al., 2013). These velocity-based mixing indices are more suited to purely hydrodynamic studies where the fluid studied is homogeneous but the aim is to perturb the fluid flow profile (Bamieh et al., 2001). This is often a precursor to more complicated heat and mass transport studies where the aim is to decrease inhomogeneity in the concentration and/or temperature

* Corresponding author.
E-mail address: J.Bao@unsw.edu.au (J. Bao).

http://dx.doi.org/10.1016/j.ces.2016.09.013

Nomenclature

Symbols

- A — state space matrix that relates rate of change in $\mathbf{X}(t)$ to the present value of $\mathbf{X}(t)$
- A_m — state space matrix that relates rate of change in $\mathbf{X}_m(t)$ to the present value of $\mathbf{X}_m(t)$. Changes for each wavenumber m
- B — state space matrix that relates the rate of change in $\mathbf{X}(t)$ to the rate of change in $\mathbf{w}(t)$
- B_m — state space matrix that relates the rate of change in $\mathbf{X}_m(t)$ to the rate of change in $\mathbf{w}_m(t)$. Does not change with wavenumber
- $B_{m,k}$ — complex coefficient of $e^{j\alpha_m}$ as part of the Fourier decomposition of $E_{x,k}(x)$
- C — state space matrix that converts state variables $\mathbf{X}(t)$ to output variables $\mathbf{Y}(t)$
- C_m — state space matrix that converts state variables $\mathbf{X}_m(t)$ to output variables $\mathbf{Y}_m(t)$
- $\mathbb{C}$ — set of complex numbers
- $\mathcal{D}_N$ — discrete differentiation matrix for Chebyshev collocation method with N subdivisions
- $E_{x,k}(x)$ — dimensionless electric field coefficients for the kth electrode at the slip wall surface in the x-direction
- $\hat{E}_{\hat{x}}(\hat{x}, \hat{y}, \hat{t})$ — total electric field in the streamwise direction for the real channel (kg m s^{-3} A^{-1})
- $\hat{E}_{\hat{x},k}(\hat{x}, \hat{y}, \hat{t})$ — total electric field in the streamwise direction for the real channel due to all kth electrodes (kg m s^{-3} A^{-1})
- $\hat{E}_{\hat{x},k,\theta}(\hat{x}, \hat{y}, \hat{t})$ — electric field in the streamwise direction for electrode k in the θth channel segment (kg m s^{-3} A^{-1})
- $\hat{E}_{\hat{y},k,\theta}(\hat{x}, \hat{y}, \hat{t})$ — electric field in the normal direction for electrode k in the θth channel segment (kg m s^{-3} A^{-1})
- $\bar{E}$ — matrix containing $\bar{E}_m$ for all wavenumbers m
- $\bar{E}_m$ — matrix containing the real and imaginary parts of $B_{m,k}$ for all k used for quadratic optimization
- $\bar{E}_{x,k}(x)$ — dimensionless electric field coefficients for the kth electrode at the slip wall surface in the x-direction for the quadratic optimization
- $\mathbf{E}_x^T(x)$ — dimensionless row vector of electric field coefficients at the slip wall surface in the x-direction
- $\mathbf{E}_{x,k}$ — vector of electric field at slip wall at points x_ℓ
- $\hat{\mathbf{E}}(\hat{x}, \hat{y}, \hat{t})$ — total electric field vector in real channel (kg m s^{-3} A^{-1})
- $\hat{\mathbf{E}}_k(\hat{x}, \hat{y}, \hat{t})$ — electric field vector in real channel due to all kth electrodes (kg m s^{-3} A^{-1})
- $\hat{\mathbf{E}}_{k,\theta}(\hat{x}, \hat{y}, \hat{t})$ — electric field vector for electrode k in the θth channel segment (kg m s^{-3} A^{-1})
- $\hat{\mathbf{E}}_{\hat{x}}^T(\hat{x})$ — row vector of electric fields in the $\hat{x}$-direction for the real channel (kg m s^{-3} A^{-1})
- $G(j\omega)$ — frequency response from $\tilde{\mathbf{u}}(j\omega)$ to $\tilde{\mathbf{Y}}(j\omega)$
- $\mathcal{I}$ — identity matrix. Subscript indicates size of matrix
- $\mathcal{I}_a$ — matrix converting $\bar{\mathbf{a}}_m^R(t)$ to $\mathbf{a}_m(t)$
- $J(\mathbf{u}(t), T)$ — time domain objective function
- $J'(\underline{\tilde{\mathbf{u}}}(j\omega_0))$ — frequency domain objective function
- $J_{,c}(\underline{\tilde{\mathbf{u}}}(j\omega_0))$ — frequency domain objective function for the system with input constraints
- $J_{,u}(\underline{\tilde{\mathbf{u}}}(j\omega_0))$ — frequency domain objective function for the unconstrained system
- L — dimensionless length of channel
- L_k — location of the centre of electrode k in the x-direction of the dimensionless channel
- $\hat{L}$ — length of the real channel (m)
- $\hat{L}_i, \hat{L}_j, \hat{L}_k$ — location of centre of electrode i, j, and k respectively in the streamwise direction for the real channel (m)
- M — largest wavenumber considered in the study
- N — positive integer number of subdivisions in the y-direction
- $P_m(x)$ — complex exponential with m oscillations per dimensionless length L
- $Q(\gamma(j\omega_0))$ — function that forces the sum of charges to be zero when solved for $\gamma(j\omega_0)$
- $\bar{Q}(\gamma(j\omega_0))$ — simplified version of $Q(\gamma(j\omega_0))$ that obtains only real solutions to the overall problem
- R — positive integer number of solutions to $\bar{Q}(\gamma(j\omega_0))$
- $\bar{R}$ — matrix converting $\mathbf{a}_m(t)$ to $\bar{\mathbf{a}}_m^R(t)$
- $\mathbb{R}$ — set of real numbers
- Re — Reynolds number
- T — period of oscillation
- $\hat{U}$ — characteristic velocity of fluid in the real channel (average crossflow velocity) (m s^{-1})
- $\tilde{U}(y)$ — steady state crossflow profile for the dimensionless channel
- $\tilde{U}_N^{(q)}$ — diagonal matrix where each diagonal entry is qth derivative of $\tilde{U}(y)$ evaluated at each y_n
- $\mathcal{U}$ — space of voltages $\mathbf{u}(t)$ such that $\mathbf{u}(t)$ satisfies all necessary constraints
- $\mathcal{U}_{\mathcal{F}}$ — space of voltages $\underline{\tilde{\mathbf{u}}}(j\omega)$ such that $\underline{\tilde{\mathbf{u}}}(j\omega)$ satisfies all necessary constraints
- $V(j\omega)$ — orthonormal input matrix in singular value decomposition
- $\tilde{V}_k(j\omega_0)$ — kth component of $\tilde{V}(j\omega)$
- $\tilde{V}_k^I(j\omega_0)$ — imaginary part of $\tilde{V}_k(j\omega)$
- $\tilde{V}_k^R(j\omega_0)$ — real part of $\tilde{V}_k(j\omega)$
- $V_{i,j}(t)$ — dimensionless potential difference between electrodes i and j
- $\hat{V}(\hat{x}, \hat{y}, \hat{t})$ — real electric potential as a function of location and time (kg m^2 s^{-3} A^{-1})
- $\hat{V}_i(\hat{t}), \hat{V}_j(\hat{t})$ — real electric potential at the top of electrode i and j respectively (kg m^2 s^{-3} A^{-1})
- $\hat{V}_{i,j}(\hat{t})$ — real potential difference between electrodes i and j (kg m^2 s^{-3} A^{-1})
- $\hat{V}_{i,j,r_{\max}}(\hat{t})$ — real potential difference between electrodes i and j for the most effective input (kg m^2 s^{-3} A^{-1})
- $\underline{\tilde{V}}_{i,j}(j\omega_0)$ — complex coefficient of complex exponential of dimensionless potential difference between electrodes i and j
- $\underline{\tilde{V}}_{i,j}^R(j\omega_0)$ — real coefficient of cosine wave of dimensionless potential difference between electrodes i and j
- $\underline{\tilde{V}}_{i,j}^I(j\omega_0)$ — real coefficient of sine wave of dimensionless potential difference between electrodes i and j
- W_y — weighting function applied in C_m such that $W_y^T W_y = W_y^H$
- $W_y{}^H$ — symmetric weighting function such that it satisfies weightings necessary for an appropriate quadrature
- $X(j\omega)$ — orthonormal output matrix in singular value decomposition
- $\mathbf{X}(t)$ — state vector containing coefficients at all wavenumbers
- $\mathbf{X}_m(t)$ — state vector containing coefficients at wavenumber m
- $Y(x, y, t)$ — pseudo-output for velocity magnitude field
- $\mathbf{Y}(t)$ — pseudo-output vector containing velocity profile at each Chebyshev–Gauss–Labotto point
- $\mathbf{Y}(x, t)$ — output vector of velocity profiles at each Chebyshev–Gauss–Labotto point
- $\mathbf{Y}_m(t)$ — output vector containing coefficients at wavenumber m