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Turbulent spectrum model for drop-breakup mechanisms in an inhomogeneous turbulent flow

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ABSTRACT

Drop-breakup models for liquid/liquid dispersion in turbulent flow mostly derive from Kolmogorov and Hinze analysis. They are of a semi-empirical type, since they are based on power laws involving at least Weber and Reynolds numbers. The pre-factors are determined by fitting the experimental data on droplet diameters in the various configurations. The main cause for the discrepancies in the fitting constant between different flow types is the intrinsic spatial heterogeneity of the turbulent field, especially the turbulent kinetic energy (TKE) dissipation rate ε. This feature explains why there is no universal physical model suitable for breakup prediction throughout the range of flow geometries.

In the present work, we investigate the drop size distribution by reference to Hinze's actual theory in a local approach, attempting a direct interpretation thanks to the turbulence spectra measured (by LDA) in the most dissipative locations of a flow which is basically inhomogeneous. This method allows estimation of droplet size with no constant fitting and with acceptable accuracy; however the knowledge of the turbulence field is required. The present study is carried out with low dispersed-phase fraction, so that the coalescence is negligible. Experiments show that the "typical value" for the TKE dissipation rate to fit the raw model of Hinze and Kolmogorov lies between the maximum and the mean value in the flow field. The issue of "typical ε value" hence avoided is discussed by physical arguments for the flow structure.

1. Liquid/liquid dispersion theories

The study of liquid/liquid or gas/liquid dispersion is of great interest in a wide range of industries, among them chemical engineering applications involving interfacial reactions and fluid mixing in the food, cosmetics, pharmaceutics, and paper industries. In all these systems, process efficiency is governed by droplet size, and this has motivated extensive research on bubble or droplet breakup. Taylor (1932, 1934) established the theory of droplet deformation in the laminar regime that is briefly described below, although our study deals with the turbulent regime. He based his analysis on a simple force balance stating that the breakup is governed by a dimensionless group Ca (the capillary number) that represents the ratio of the deforming external viscous stress of the laminar flow and the resisting Laplace forces (half of them):

$$Ca = \frac{\tau\, d}{2\,\sigma} \tag{1}$$

where σ is the surface tension, d the droplet diameter and τ the external viscous stress defined in Eq. (2), where μ_c is the continuous-phase viscosity and S the strain-rate tensor:

$$\tau = \mu_c \sqrt{2S{:}\,S} \tag{2}$$

The generalized shear rate is defined by

$$\dot{\gamma} = \sqrt{2S{:}\,S} \tag{3}$$

Breakup is expected to occur when the value of the dimensionless group Ca exceeds a critical value Ca_{crit} that depends on the relative phase viscosity ratio p (the parameter describing the resistive internal viscous forces):

$$p = \frac{\mu_d}{\mu_c} \tag{4}$$

where μ_d is the dispersed phase viscosity. The dependence of drop deformation on p was investigated experimentally and theoretically by resolving the Stokes equations around the drop, as thoroughly de-

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scribed by Grace (1982) for shear and extensional flows. In fact, the critical ratio appears to depend also on the vorticity (extensional component). The maximum bubble size $d_{\max}$ that can "survive" in the flow is given by:

$$Ca_{crit} = \frac{\mu_c \dot{\gamma} d_{\max}}{2\sigma} \tag{5}$$

with the value Ca_{crit} depending on p and its minimum is around 0.5 for $p = 1$. Fig. 1 shows the behavior of Ca_{crit} throughout the range of p for both simple shear flow (Couette flow) and extensional flow (four-roll hyperbolic flow). An important feature is that the shear flow cannot achieve breakup for a dispersed-phase viscosity greater than four times the continuous-phase viscosity, whereas this limit does not exist when there is a significant extensional component. Breakup also appears to be more efficient in the latter case as the critical Capillary number is lower throughout the whole range of p (Bentley and Leal, 1986). The diagram reproduced on Fig. 1 allows determination of the maximum drop diameter that can be encountered in the flow, but not the size distribution resulting from the stochastic breakup process resulting from the global flow pattern. Nevertheless, we believe, like Ottino *et al.* (2000) that "understanding of this diagram constitutes the minimum knowledge needed to rationalize dispersion process in complex flows".

This argument can be transposed to the turbulent regime, but the relevant stress for the breakup phenomenon is here the inertial forces of the turbulent field. The fundamental theory for bubble or drop breakup in the literature was suggested independently by Kolmogorov (1949) and Hinze (1955). The Hinze-Kolmogorov theory is based on two assumptions: (i) in homogeneous and isotropic turbulence, inertial forces in the turbulent eddies are more intense than viscous forces and are responsible for breakup; (ii) only velocity fluctuations at the drop-diameter scale can cause the large deformations required for the drop breakup. Of course, these assumptions are applicable when the range of drop diameters falls in the inertial domain of the turbulence spectrum, meaning in the turbulent cascade length scales. It can be noticed that for droplets of diameters below the Kolmogorov length scale, the breakup is essentially governed by viscous shear, $\dot{\gamma}_{vis} = (\frac{\varepsilon}{\nu})^{1/2}$, and the Taylor-Grace theory is relevant for predicting the scale of the droplet size.

Returning to the cascade domain, the stress τ responsible for drop deformation is based on the Reynolds tensor for eddies of size d:

$$\tau = \rho \overline{\delta u^2(d)} \tag{6}$$

where ρ is the density of the continuous phase and $\overline{\delta u^2(d)}$ is the longitudinal autocorrelation over a distance d equal to the drop diameter, which is the second-order structure function $S_2(d)$ in wavelength space (Monin and Yaglom, 1975):

$$\overline{\delta u^2(d)} = S_2(d) = \overline{u^2(x+d) - u^2(x)} \tag{7}$$

Therefore the expression for the drop Weber number, equivalent to a turbulent capillary number, is given by:

$$We = \frac{\rho \overline{\delta u^2(d)}\, d}{\sigma} \tag{8}$$

If d is within the inertial range of the turbulent scales, $S_2(d)$ can be expressed from the energy cascade theory for equilibrium turbulence by:

$$S_2(d) = \overline{\delta u^2(d)} = \beta \varepsilon^{2/3} d^{2/3} \tag{9}$$

where ε is the local turbulent kinetic energy dissipation rate (Batchelor, 1953). Eq. (9) is the Kolmogorov two-thirds law and β is called the Kolmogorov constant in the physical space, a "true" constant for homogeneous and isotropic turbulence (often denoted by C_K or C_2 in the literature). Hence the final expression for the drop Weber number is:

$$We = \frac{\rho \beta \varepsilon^{2/3} d^{5/3}}{\sigma} \tag{10}$$

The force balance implies the existence of a critical value above which breakup can occur, We_{crit}, and then the maximal drop diameter can be predicted by:

$$d_{\max} = \left(\frac{We^{crit}}{\beta}\right)^{3/5} \left(\frac{\sigma}{\rho}\right)^{3/5} \varepsilon^{-2/5} \tag{11}$$

which can be written

$$d_{\max} = \left(\frac{We_{crit}}{\beta\, \varepsilon^{2/3}}\right)^{3/5} \left(\frac{\sigma}{\rho}\right)^{3/5} \tag{12}$$

The proportionality of $d_{\max}$ to $(\sigma/\rho)^{3/5}\varepsilon^{-2/5}$ in Eq. (11), referred as the HK (Hinze-Kolmogorov) model, has been validated in several studies to fit the maximum drop size, but the values of the pre-factor $(We_{crit}/\beta)^{3/5}$ and We_{crit} are not generally agreed upon and not very constant: some examples from the literature are given in Table 1.

The value of the critical Weber number is open for discussion: depending on the drop breakup mode – binary (Xiang et al., 2011; Irannejad and Jaberi, 2014), ternary (Konno et al., 1983) – there are probably slightly different critical values. Nevertheless, the various values of this parameter obtained theoretically or by simulation (Hesketh et al., 1991) in liquid/liquid dispersions are of order of unity. In this study We_{crit} is then fixed at unity in accordance with the analysis of Hinze (1955).

The discrepancies observed in the $(We_{crit}/\beta)^{3/5}$ values in Table 1 may also be explained by the β values, which may depend on the local turbulent structure and cannot easily be attained experimentally. Authors generally estimate this value from data in the literature for similar flows (Martinez-Bazan et al., 1999a). Sometimes it is explicitly fixed to fit the experimental drop size (Zaccone et al., 2007).

Another significant difficulty arises in the adequate choice of the ε value in the HK model, which is defined for local properties in an inhomogeneous turbulence, unlike the dispersion result, which is observed as a global process. In real industrial cases, sharp gradients exist for all the turbulent quantities, so that the reference ε value becomes very arbitrary. The more usual choice is then the mean value, since that can be easily determined by the energy balance in the process. Alternative options can be mentioned: for instance, the maximum ε value "seen" by the fluid particles in the flow can be considered more suitable to scale the maximum diameter. The latter idea is relevant if the residence time is sufficient to ensure that the particles stay "long enough" in the higher-turbulence region in the flow

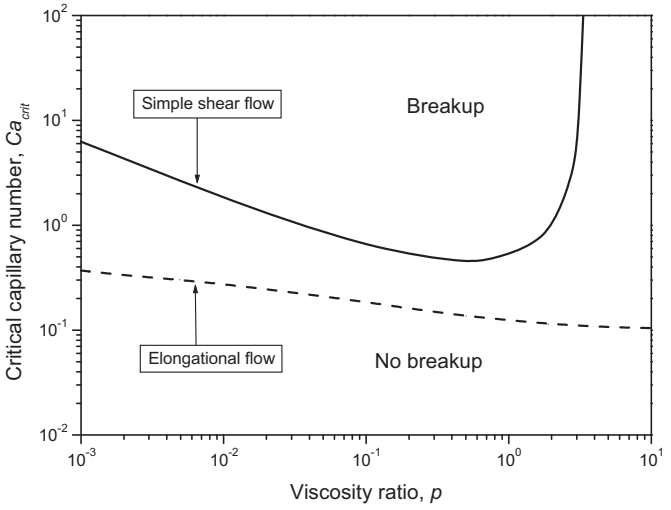

Fig. 1. Critical capillary number.
Adapted from Grace (1982).