Combustion and Flame 173 (2016) 99–105

Contents lists available at ScienceDirect

Combustion and Flame

journal homepage: www.elsevier.com/locate/combustflame

Theoretical and numerical analysis of oscillating diffusion flames

Milan Miklavčič [a,*], Indrek S. Wichman [b]

[a] Department of Mathematics, Michigan State University, East Lansing, MI 48824, USA
[b] Energy and Automotive Research Laboratories (EARL), 1497 Engineering Research Court, Department of Mechanical Engineering, Michigan State University, East Lansing, MI 48824, USA

ARTICLE INFO

Article history:
Received 16 March 2016
Revised 26 August 2016
Accepted 27 August 2016
Available online 11 September 2016

Keywords:
Diffusion flames
Oscillations
Coflow slot burners
Theoretical analysis
Burke–Schumann model
Infinite-rate chemistry

ABSTRACT

A novel method is presented for solving the forced transient diffusion flame in the exit region of a coflow burner. Streamwise diffusion is eliminated, which produces the Burke–Schumann model. A mathematical transformation renders the transient, forced convection problem equivalent to a steady-state convection problem. The transformation differs from previous approaches because its use does not require a priori restriction to small perturbations. For this reason, flow fluctuations that are large fractions of the initial flow field may be described exactly and features of nonlinear response can be examined without recourse to detailed numerical simulation. The method is applied to study flame evolution and oscillation for two physically separated coflow slot burner flames as they merge.

1. Introduction

In the study of diffusion flames, the Burke–Schumann (B–S) flame model figures prominently as seen in the extensive discussions in [1–3]. The central feature of the B–S model, along with the restriction to infinite-rate chemistry, is the neglect of streamwise diffusion of species, thermal energy and momentum for burner-attached flames. This restriction means that the B–S formulation is a boundary-layer formulation [4]. The neglect of the streamwise component of diffusion renders the problem simpler and easier to solve. This approach has been employed to study many shapes and arrangements of diffusion flames in the science and technology of burner development.

Simplified analytical models provide important physical and mathematical information in the form of quantifiable relationships, which pure numerical models cannot. This is perhaps the principal reason that the studies of Roper [5,6] continue to be referenced in the combustion literature at an unabated rate. In addition, it is the principal reason that numerical simulations of oscillating diffusion flames are discussed and interpreted in terms of simplified theoretical models. The predictions of the former, though limited by their restrictions (e.g., constant properties, infinite-rate chemistry) and simplifications, allow for the interpretation of the solutions of the full equations with variable-properties, multiple-step kinetics, and complex diffusion.

In the past two or three decades, hundreds if not thousands of studies and laboratory trials have been carried out on burner flames.

One important area where diffusion flame oscillations play a role is in the general topic of burner flame attachment. Here, the challenge is to describe the means by which a diffusion flame attaches itself when fuel (or oxidizer) flows through a tube that is surrounded by—or adjacent to—the opposite reactant, namely oxidizer (or fuel). The classical version of this problem [3] describes a jet of fuel ejected into a surrounding, quiescent oxidizer. In this case, the problem may involve turbulence (given a sufficiently large inlet reactant jet Reynolds number) and the associated development of localized shear layers. In order to avoid the fluid mechanical complications caused by vortex trains and coherent structures interlaced with small, intense vortices, researchers have examined jets in which the fuel and surrounding oxidizer have similar velocities. Items of interest are flame heights, combustion rates, flame standoff distances and conditions producing blowoff. Fundamental studies on this topic have been conducted over many years by Chung and co-workers [7,8].

Recent work on oscillating diffusion flames has examined the response of flames to both axial velocity and mixture fraction oscillations [9]. In this work, the mixture fraction equation is solved using perturbation analyses [10] that posit a basic state subjected to flow perturbations. The basic restriction in [9] is to *small* flow perturbations. Thus, the linearized analysis is handicapped because

* Corresponding author.
E-mail addresses: milan@math.msu.edu (M. Miklavčič), wichman@egr.msu.edu (I.S. Wichman).

http://dx.doi.org/10.1016/j.combustflame.2016.08.023
0010-2180/

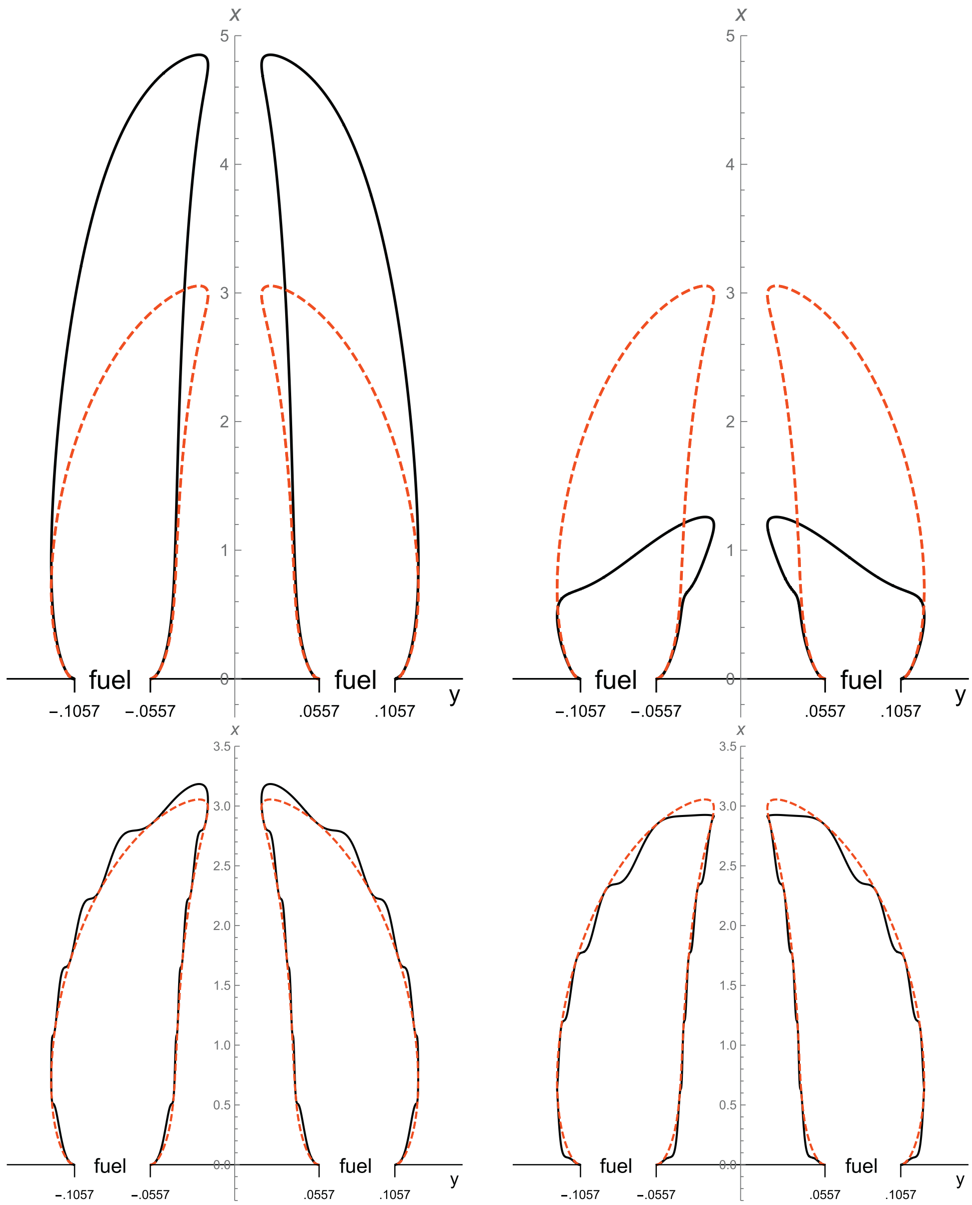

Fig. 1. The burners of width 0.05 are separated by 2 × 0.0557 and the fuel is flowing in x-direction through the fuel slots with the average bulk velocity $u_0 = 100$. The stationary flame corresponding to the average bulk fuel velocity is represented by a dashed line. The bulk fuel velocity is oscillating with amplitude $\varepsilon = 0.9 = 90\%$ of the average velocity u_0 and extrema of the oscillating flame boundaries are represented by solid curves. The circular frequency of oscillations is $\omega = 100$ for the top pair. $\omega = 1100$ for the bottom pair. The diffusion constant is $D = 0.1$. Units and scalings are discussed in Section 4.

the actual process is not linear. As discussed by Takahashi et al. [11], oscillations in the mixture fraction field produce oscillations in the species mixture fractions (oxidizer, fuel) which produces oscillations in the reaction rate (heat release) with resulting oscillations in the temperature field. Oscillation of the temperature field produces larger oscillations in the reaction rate because of its exponential dependence on the temperature. These nonlinear interactions make it impossible for linearized analyses to model processes such as liftoff and blowoff.

2. Problem formulation and transformation

Consider the physical configuration shown in Fig. 1. Here, fuel flows with bulk velocity u toward the positive x-direction through one or more channels that are each adjacent to one or more oxidizer channels. This configuration is called a slot burner, for which experiments have been conducted over many years by many investigators [12,13]. As is explained in detail later in Section 4, this configuration uses a Peclet number of 50, which can be attained in many ways, one of them being a flow velocity of 100 cm/s, a diffusion coefficient of 0.1 cm^2/s and a characteristic length of 0.05 cm. A Peclet number of 50, as will be explained and discussed extensively in Section 4, is entirely in line with previous experiments, hence the values indicated in Fig. 1 are grounded in empirical reality.

Subject to the restriction to equal channel exit velocities, the evolution of the mixture fraction Z for the idealized two-dimensional diffusion flame in the half-space $x \geq 0$ is given by [14,15]

$$\frac{\partial Z}{\partial t} + u\frac{\partial Z}{\partial x} = D\frac{\partial^2 Z}{\partial y^2}. \tag{1}$$

Here, diffusion in the streamwise (x) direction is assumed to be negligible in comparison with streamwise convection term, uZ_x.