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Spatially localized radiating diffusion flames

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ABSTRACT

A simple model of radiating diffusion flames considered by Kavousanakis et al. (2013) [1] is extended to two spatial dimensions. A large variety of stationary spatially localized states representing the breakup of the flame front near extinction is computed using numerical continuation. These states are organized by a global bifurcation in space that takes place at a particular value of the Damköhler number and their existence is consistent with current understanding of spatial localization in driven dissipative systems.

1. Introduction

In premixed flames, strong coupling between aerodynamics and reaction/diffusion processes arising from strong thermal expansion gives rise to Darrieus–Landau instabilities [2]. In diffusion flames, in contrast, thermal expansion plays a minor role in the thermal-diffusive processes that are behind the presence of instabilities in non-premixed flames [3]. The origin of these instabilities, which can lead to spatially uniform pulsation or to stationary or oscillating cellular structures, has been reviewed by Matalon [4,5]. Models of non-premixed flames that neglect aerodynamic effects have proved particularly useful in studies of turbulent partially premixed combustion. A well-known example is provided by the turbulent flamelet model of Peters [6] whose dynamics are dominated by diffusive processes, and not advection. The now classical flat unstrained flame introduced by Kirkbey and Schmitz [7] has been extensively studied, and numerous stability analyses [8–11] have shown that the Lewis numbers of both reactants play an essential role in selecting the nature of the possible thermal-diffusive instabilities that appear when approaching extinction (here necessarily via lean mixtures). Examples of these instabilities can be found in axisymmetric jet configurations, both experimentally [12–14] and numerically [15], as well as in tubular flames [16,17] and in unstrained flames [18,19].

The present paper is devoted to shedding additional light on the properties and dynamics of diffusion flames near flame extinction. Of particular interest in this connection is the dynamical behavior preceding extinction, including intermittency, breakup, hopping, and other types of time dependence. Diffusion flames typically exhibit cellular structures, often accompanied by temporal oscillations. Existing studies range from detailed simulations of models that retain as many of the basic processes as possible to highly simplified model systems, designed to exhibit an understanding of the qualitative properties of such flames, usually through the use of linear stability analysis in time, e.g., [20]. Such model studies are useful in developing both physical intuition and a mathematical understanding of the observed transitions. The latter often relies on bifurcation theory and relevant dynamical systems theory [1].

This paper focuses on the spatial structure of diffusion flames in the extinction regime, but goes much beyond linear stability analysis. Specifically, we show that a well-known mechanism responsible for the presence of spatially localized structures in continuous systems described by partial differential equations applies to simple models of radiating diffusion flames, and explore its consequences for the predictions of the model. This mechanism is mathematical in nature, and employs an understanding of this type of model developed using ideas based on the notion of spatial dynamics: treating the spatial profile of a stationary solution of the equations as a consequence of evolution in the spatial variable, in other words, as if space were like time [21]. This is a powerful idea that makes most sense in systems with one unbounded direction. Of course, real systems are defined by boundary-valued problems whereas time-like problems are solved with initial conditions. It turns that this difference is not crucial, and that much can be deduced based on this approach even when the domain is finite, provided it is sufficiently large. This approach is extended here to

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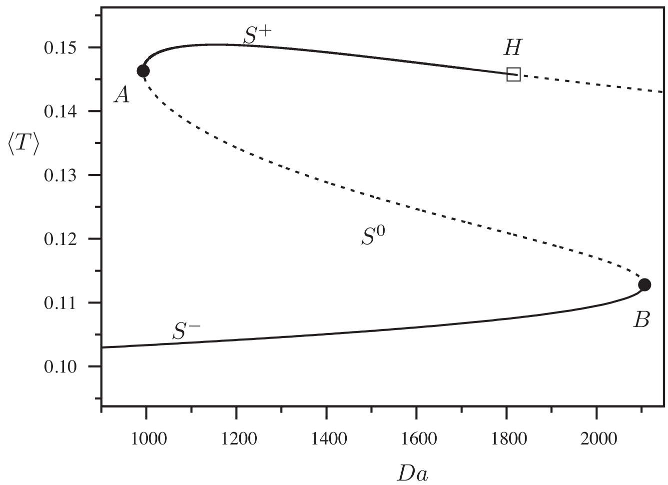

Fig. 1. The S-shaped branch of one-dimensional equilibria in terms of the average temperature $\langle T\rangle \equiv (1/2)\int_{-1}^{1}\bar{T}(x)\,dx$ as a function of the Damköhler number Da for $Le_o = Le_f = 1$, $T_b = 0.1$, $T_a = 1$ and $R = 0.2$, showing the folds A and B (solid dots) and a Hopf bifurcation at $Da \approx 1800$ (H, open square symbol) on the upper branch S^+. The labels S^0 and S^- indicate the middle and lower branches. Stable (unstable) branches are shown in solid (broken) lines.

two spatial dimensions and used to compute a large variety of stationary spatially localized states representing the breakup of the flame front near extinction. These states are organized by a global bifurcation in space that takes place at a particular value of the Damköhler number and their existence is consistent with current understanding of spatial localization in driven dissipative systems.

We consider a simple model of a radiating diffusion flame studied in [1]. Specifically, we consider a one-dimensional flame between a pair of porous walls that allow fuel (mass fraction Y_f) to diffuse in from the left ($x = -1$) and oxidizer (mass fraction Y_o) to diffuse in from the right ($x = 1$), both assumed to have the same temperature T_b. The burning process is described by a binary one step process of Arrhenius type with reaction term $w = Da\, Y_o Y_f \exp(-T_a/T)$, where T is the instantaneous temperature and the constant T_a represents the activation temperature. This temperature-activated process releases heat that is redistributed via radiation. Convection is ignored. The system is described by the nondimensional equations

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + w - R\,Da\left(T^4 - T_b^4\right), \tag{1}$$

$$Le_o\frac{\partial Y_o}{\partial t} = \frac{\partial^2 Y_o}{\partial x^2} - w, \tag{2}$$

$$Le_f\frac{\partial Y_f}{\partial t} = \frac{\partial^2 Y_f}{\partial x^2} - w \tag{3}$$

with the boundary conditions

$$T = T_b, \quad Y_f = 1, \quad Y_o = 0 \qquad \text{at} \qquad x = -1, \tag{4}$$

$$T = T_b, \quad Y_f = 0, \quad Y_o = 1 \qquad \text{at} \qquad x = +1. \tag{5}$$

Here Le_o and Le_f are the Lewis numbers of the oxidizer and fuel, respectively, R is a parameter, and Da is the Damköhler number.

Steady solutions $(\bar{T}(x), \bar{Y}_o(x), \bar{Y}_f(x))$ of this boundary value problem are independent of Le_o, Le_f and reveal the presence of the classic S-shaped response curve as a function of the Damköhler number first identified by Liñán [22] and Peters [23]. Figure 1 shows a typical result in terms of the spatial average of the temperature, $\langle T\rangle \equiv (1/2)\int_{-1}^{1}\bar{T}(x)\,dx$, plotted as a function of Da. In the following we refer to the states on the upper branch of the curve as S^+ while those on the lower branch are labeled S^-; the states in between are labeled S^0 (Fig. 1). The stability of this solution was examined in [1] for $Le_o = Le_f = 1$. The middle segment S^0 of the S-shaped response was found to be unstable, with a real eigenvalue passing through zero at both the left and right folds, labeled A and B in Fig. 1 and represented as solid circles, as expected on the basis of standard bifurcation theory. In addition, the authors identified a sequence of Hopf bifurcations on the S^+ branch, the first of which destabilizes S^+ as Da increases (square symbol in Fig. 1), leading to temporal oscillations. Other Hopf bifurcations restabilize S^+ at larger Da (not shown), so that in all cases the large Da part of S^+ was found to be stable. This is of course the ignited state. Thus the results of [1] can be interpreted as showing that, within this model at least and for appropriate parameter values, the flame undergoes oscillations prior to extinction as Da decreases.

In this paper, we are interested in the steady solutions that bifurcate from the folds A and B on the S-shaped branch when the problem is extended to the (x, y) plane, $-\infty < y < \infty$. We refer to the y direction as the transverse direction. In other words, we study steady solutions of the problem

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + w - R\,Da\left(T^4 - T_b^4\right), \tag{6}$$

$$Le_o\frac{\partial Y_o}{\partial t} = \frac{\partial^2 Y_o}{\partial x^2} + \frac{\partial^2 Y_o}{\partial y^2} - w, \tag{7}$$

$$Le_f\frac{\partial Y_f}{\partial t} = \frac{\partial^2 Y_f}{\partial x^2} + \frac{\partial^2 Y_f}{\partial y^2} - w \tag{8}$$

with the y-independent boundary conditions (4)–(5) on $x = \pm 1$ and periodic boundary conditions in y with a large spatial period $L_y \gg 1$. The resulting system is spatially reversible, i.e., it is invariant under the transformation $y \to -y$. This is an important property of the model that has important consequences for the spatial eigenvalues of the base (y-invariant) state [21].

As in the one-dimensional case steady solutions of this problem are necessarily independent of the Lewis numbers Le_o, Le_f. Moreover, one class of solutions consists of those found in [1], extended uniformly in y. However, when the stability of these solutions is examined with respect to y-dependent perturbations one finds that other, y-dependent, solutions may be present. This is so despite the fact that there is no bifurcation to periodic states in the y direction, the so-called striped flames. This fact can be established by examining the linear problem for an infinitesimal perturbation $(a'(x, y, t), b'(x, y, t), c'(x, y, t))$ of the one-dimensional time-independent solution $(\bar{T}(x), \bar{Y}_o(x), \bar{Y}_f(x))$. This linear problem can be separated by writing $(a'(x, y, t), b'(x, y, t), c'(x, y, t)) = (a(x), b(x), c(x))\exp(\sigma t + iky)$. The resulting k-dependent linear problem has no solutions with zero growth rate σ when $k \neq 0$, indicating the absence of a pattern-forming Turing instability. However, when $k = 0$ the linear problem does have two locations where $\sigma = 0$. These are precisely the folds A and B on the S-shaped branch, where – as already mentioned – the stability problem for the one-dimensional solution $(\bar{T}(x), \bar{Y}_o(x), \bar{Y}_f(x))$ necessarily has a zero eigenvalue.

The paper is organized as follows. In Section 2, we provide a brief summary of the essential input from dynamical systems theory that guides our study. Detailed results are presented in Section 3 and these confirm the bifurcation structure anticipated in Section 2. The paper concludes with a brief discussion and conclusions. The numerical procedure used to compute localized structures in the present system is described in the Appendix.

2. A brief review of the theory

The theory summarized below applies to systems with a single unbounded direction such as the y direction in the present problem. We suppose that the system exhibits an S-shaped branch