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Development of local dynamic mode decomposition with control: Application to model predictive control of hydraulic fracturing

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ABSTRACT

Dynamic mode decomposition with control (DMDc) is a modal decomposition method that extracts dynamically relevant spatial structures disambiguating between the underlying dynamics and the effects of actuation. In this work, we extend the concepts of DMDc to better capture the local dynamics associated with highly nonlinear processes and develop temporally local reduced-order models that accurately describe the fully-resolved data. In this context, we first partition the data into clusters using a Mixed Integer Nonlinear Programming based optimization algorithm, the Global Optimum Search, which incorporates an added feature of predicting the optimal number of clusters. Next, we compute the reduced-order models tailored to each cluster by applying DMDc within each cluster. The developed models are subsequently used to compute approximate solutions to the original high-dimensional system and to design a feedback control system of hydraulic fracturing processes for the computation of optimal pumping schedules.

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1. Introduction

Many chemical processes and fluid systems have models that apparently describe their dynamics to near-perfect accuracy. However, very often, these turn out to be high-dimensional complex models and the fully-resolved simulations necessary to capture their detailed nonlinear behaviors put a considerable strain on computational resources. This limits the capability to perform parameter estimation or design feedback control systems which require real-time computation of dynamic solutions. Nevertheless, despite the fact that they are governed by high-dimensional systems, very often the dominant behavior can be captured by modes that are many orders of magnitude smaller than the dimension of the original system. For instance, Noack et al. (2003) showed that as few as three ordinary differential equations can describe the essential features of a laminar flow past a 2D cylinder. Thus, practical engineering strategies for dealing with high-dimensional data require developing simplified (reduced-order) models that significantly reduce the dimension of the underlying system to remain computationally-tractable at the negligible expense of model accuracy.

The field of reduced-order modeling is large, and new methods are being developed at a fast pace. Among these, two of the most commonly used modal decomposition techniques are Proper Orthogonal Decomposition (POD) and Dynamic Mode Decomposition (DMD). Both of them are based on analyzing information from a sequence of observational data arising from high-dimensional systems to identify coherent structures embedded in the system. POD determines the structures that capture the most energy to optimally reconstruct a data set arising from a linear or nonlinear dynamical process in the mean square sense (Holmes et al., 1996). However, the energy criterion may not be the relevant measure to precisely rank the flow structures in all the circumstances (Noack et al., 2008). DMD has originally been introduced in the fluids community (Schmid and Sesterhenn, 2008) to yield flow structures that accurately describe the motion of the flow. In contrast to POD, this method extracts modes that are dynamically relevant spatial structures rather than selecting the dominant modes that capture most of the flow energy. Ghommem et al. (2014) has shown that the DMD-based approach extracted modes that are more relevant for long-term dynamics compared to POD. Computationally, DMD assumes a linear model that best represents the underlying dynamics, even if those dynamics stem from a nonlinear process. Although it might seem equivocal describing a nonlinear system by superposition of modes whose dynamics are governed by the corresponding eigenvalues, DMD can be considered a numerical

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approximation to Koopman spectral analysis providing theoretical justification for characterizing nonlinear systems (Rowley et al., 2009; Mezić, 2013; Bagheri, 2013). After gaining quick popularity, DMD has found successful implementation in many fluid mechanics applications to analyze both numerical (Schmid, 2010; Schmid et al., 2011; Seena and Sung, 2011; Mizuno et al., 2011; Muld et al., 2012) as well as experimental (Schmid, 2009, 2011; Schmid et al., 2009; Pan et al., 2011; Semeraro et al., 2012; Lusseyran et al., 2011; Duke et al., 2012) flow field data and help characterizing relevant physical mechanisms. Several efforts have been made to explore the connections of DMD with other methods, such as Eigensystem Realization Algorithm (ERA) (Tu et al., 2014), Fourier Analysis (Chen et al., 2012), POD (Schmid, 2010) and Koopman analysis (Rowley et al., 2009; Mezić, 2013; Bagheri, 2013). Several variants of the DMD algorithm have also been proposed, including optimized DMD (Chen et al., 2012), optimal mode decomposition (Goulart et al., 2012; Wynn et al., 2013), sparsity promoting DMD (Jovanović et al., 2014) and extended DMD (Williams et al., 2015).

Within this context, Proctor et al. (2016) extended the concepts of DMD and introduced the method of Dynamic Mode Decomposition with control (DMDc) to utilize both measurements of the system and applied external inputs in extracting the underlying dynamics. Additionally, DMDc also provides a description of how the control inputs affect the system, and with this understanding of the input-to-output behavior, a reduced-order model can be generated and used in the design of feedback control systems to regulate the original high-dimensional systems. DMDc inherits a number of advantages of DMD in that it is a completely data-driven framework and can be applied to nonlinear systems. Furthermore, there are a number of connections between DMDc and other popular system identification methods such as Observer Kalman Filter Identification (OKID) (Juang et al., 1991), Numerical Algorithms for Subspace State Space System Identification (N4SID) (Van Overschee and De Moor, 1994), Multivariable Output Error State Space (MOESP) (Van Overschee and De Moor, 1996), and Canonical Variate Analysis (CVA) (Katayama, 2005). Algorithmically, these methods involve regression, model reduction, and parameter estimation steps similar to DMDc. However, differences do exist in terms of the similarity transformation required for projection and the use of an orthogonal complement of control inputs to generate the approximate solution (Qin, 2006). Therefore, DMDc can be used in diverse engineering applications, one of which is presented in this manuscript, where the study of dynamics while simultaneously considering the applied control input to the complex systems is important.

However, for a highly nonlinear system, the assumption of linear relation might not work well, especially in the case of limited access to spatial and/or temporal measurements. Additionally, these global methods fail to capture the local dynamics when the process parameters change with space and time (e.g., permeability and Young's modulus in the rock formation are space-dependent constants). Based on these observations, in order to capture the local dynamics of a complex nonlinear system more effectively, the embedded coherent structures must be tailored to the temporally local behavior of every portion of the solution trajectory. This idea of local model reduction has been applied successfully in several applications. In Dihlmann et al. (2011), the time domain was partitioned into multiple subdomains and temporally local eigenfunctions were used to construct a reduced-order model. Local bases were also exploited by Anttonen (2001) in the context of aeroelastic applications according to a space domain partition. In Ghommem et al. (2013), a global-local model reduction approach was presented where a generalized multiscale finite element method (GMsFEM) was combined with DMD and/or POD and applied to flows in high-contrast porous media. In Efendiev et al. (2012), a balanced truncation based global model reduction was efficiently combined with the local model reduction tools introduced in Efendiev et al. (2011). Our previous work also exploited the idea of time-domain partitioning to develop temporally local reduced-order models to accurately approximate the fully-resolved data (Narasingam et al., 2017).

Our contribution in this work is integrating the idea of temporal-clustering to DMDc to develop tailored temporally local reduced-order models that reproduce the essential features of the underlying system and quantify the effect of control inputs on the process dynamics of the system. Because the accuracy of the local approach depends on the number of clusters, our motivation is to obtain inexpensive decomposition problems while preserving the accuracy of the approximated models. In the proposed framework, we achieve this goal by using the Global Optimum Search algorithm (GOS) which predicts the optimal number of clusters based on the similarity and dissimilarity of the cluster configurations (Tan et al., 2007). Since the developed temporally local reduced-order models are computationally inexpensive, yet fairly accurate dynamic models, they can be used to design feedback control systems based on a model predictive control (MPC) framework. The proposed model reduction technique is different from other earlier works in that it is a completely data-driven approach and does not require any knowledge in terms of the system model. For example, Christofides and Daoutidis (1997) used an elegant singular perturbation based approximate inertial manifolds (AIMs) for the construction of low-dimensional ordinary differential equations (ODEs) that are subsequently used in the synthesis of non-linear output feedback controllers that guarantee stability. Baker and Christofides (2000) extended this concept to nonlinear parabolic PDE systems by first computing a set of empirical eigenfunctions to be used as basis functions in the Galerkin's framework. In Armaou and Christofides (2001), a mathematical transformation was used to construct a finite-dimensional model of a nonlinear parabolic PDE system with time-varying spatial domains. Izadi and Dubljevic (2013) presented a systematic approach to compute time-varying empirical eigenfunctions by an appropriate mapping onto the time-varying spatial domain. However, the aforementioned methods require mathematical expressions for system models and their time-varying spatial domains to construct low-dimensional approximate models. In contrast, the proposed approach requires no such knowledge and approximate models can simply be obtained using the data snapshots.

The remainder of this work is organized as follows. Section 2 provides a theoretical background on DMD and presents mathematical formulations for DMDc. Section 3 discusses the proposed methodology and outlines an algorithm for computing temporally local reduced-order models. Application of the proposed temporally local DMDc method on a hydraulic fracturing process is described in Section 4, and a series of numerical simulation results including (a) a performance assessment of the proposed local method over the global method, (b) a study of factors influencing the accuracy of the proposed method and (c) a comprehensive design of a model predictive controller to achieve a specified control objective in the hydraulic fracturing process are presented.

2. Theoretical background

2.1. Dynamic mode decomposition

DMD is a method that can extract dynamically relevant spatial structures solely from the data of a high-dimensional complex system. These structures, called dynamic modes, are equivalent to a linear tangent approximation and describe the dominant dynamic behavior of a nonlinear data sequence. In this section, we present a short mathematical description of DMD and the formulations required for DMDc.