Computers and Chemical Engineering 95 (2016) 10–20

Contents lists available at ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

Economic model predictive control of chemical processes with parameter uncertainty

Omar Santander, Ali Elkamel, Hector Budman*

Chemical Engineering Department, University of Waterloo, Waterloo, ON N2L3G1, Canada

ARTICLE INFO

Article history:
Received 7 March 2016
Received in revised form 20 August 2016
Accepted 23 August 2016
Available online 27 August 2016

Keywords:
Real time optimization
Economic nonlinear predictive control

ABSTRACT

This work proposes an EMPC (Economic Model Predictive Control) algorithm that integrates RTO (Real Time Optimization) and EMPC objectives within a single optimization calculation. Robust stability conditions are enforced on line through a set of constraints within the optimization problem.

A particular feature of this algorithm is that it constantly calculates a set point with respect to which stability is ensured by the aforementioned constraints while searching for economic optimality over the horizon. In contrast to other algorithms reported in the literature, the proposed algorithm does not require terminal constraints or penalty terms on deviations from fixed set points that may lead to conservatism.

Changes in model parameters over time are also compensated for through parameter updating. The latter is accomplished by including the parameters' values as additional decision variables within the optimization problem.

Several case studies are presented to demonstrate the algorithm's performance.

1. Introduction

Chemical Plants are designed with the task of transforming raw materials into more valuable products. These transformations must occur in the most efficient way in order to attain different goals such as maximization of product yield, minimization of the amount of contaminants or by-products, minimization of the energy employed in the process etc. Furthermore, these transformations have to be carried out under economical, physical and environmental constraints and they must be robust to variations in process settings like temperature, input flows and pressures or variations in raw material quality. To achieve these goals advanced model based controllers such as MPC are widely used since they can optimally deal with multivariable interactions while accounting for process constraints.

The conventional hierarchical control structure (see (Findeisen et al., 1980; Luyben et al., 1990)) implemented in most process industries involves an RTO (real time optimization) (Naysmith and Douglas, 1995) level above a multivariate control level realized by an MPC or other multivariable control strategy followed by lower level single-input single-output controllers (e.g. PIDs) to effect control of actuators. The RTO is generally executed to maximize a steady state economic cost with respect to steady state values of process variables that are used as set points in the lower level multivariable control strategy. Thus, the RTO provides targets (set-points) and the multivariable controller (e.g. MPC) controls the system around these targets. Although this hierarchical strategy has resulted in good performance in industrial applications there is an opportunity for improvement since chemical processes are rarely at steady state. Hence, the steady state set points calculated by the RTO and enforced by the MPC controller may not be optimal during transient scenarios.

There are several additional drawbacks related to this two layer structure. Often the RTO and MPC layers employ different models, with RTO commonly using a detailed steady state model whereas MPC generally uses simplified dynamic models which steady state values may not exactly match those calculated by the RTO algorithm. Hence the set points computed by the RTO may be sometimes unreachable by the MPC layer. Moreover, the frequency of calculation is typically different for the two layers: MPC is optimized at every sampling period whereas RTO is optimized once a new steady state has been reached. Thus, the RTO's sampling period is typically in the order of hours or even days whereas for MPC it is in the order of minutes-seconds (Ellis et al., 2014). Since industrial processes are subjected to continuous disturbances the process may never reach a steady state.

The fact that the steady state does not always correspond to the optimal economic operation (Budman and Silveston, 2008; Huang

* Corresponding author.
E-mail address: hbudman@uwaterloo.ca (H. Budman).

http://dx.doi.org/10.1016/j.compchemeng.2016.08.010

et al., 2011, 2012; Limon et al., 2014; Budman et al., 1996) has motivated Economic Model Predictive Control (EMPC) (Ellis and Christofidies, 2013, 2014; Angeli et al., 2012). EMPC maintains many of the strengths of MPC such as the use of dynamic MIMO models, the explicit handling of constraints, feedback etc (Morari and Lee, 1999; Rawlings, 2000; Grune and Pannek, 2011). However, in contrast with conventional MPC, it directly optimizes an economic cost instead of the typical quadratic stage cost that penalizes tracking errors with respect to set-points in controlled and manipulated variables. To ensure stability most EMPC algorithms previously reported (Amrit et al., 2011; Diehl et al., 2011; Angeli et al., 2009; Rawlings et al., 2012) used terminal constraints based on a particular steady state value but these may lead to conservative results. The need to avoid terminal constraints to reduce conservatism has been identified by Heidarinejad et al. (Heidarinejad et al., 2012) that proposed a two-stage algorithm to control and optimize the system in each stage respectively. In addition to the economic cost, most previously reported EMPC methods used tracking terms in the objective function that penalize deviations in controlled and manipulated variables with respect to the chosen steady state. The calculation of the steady state has to be done off-line by the RTO level. Also, robustness of EMPC to bounded disturbances has been studied in (Heidarinejad et al., 2012) but robustness to model variations has not been explicitly studied.

In this work we propose a robust nonlinear EMPC algorithm. The motivation was to propose an EMPC algorithm that avoids some of the assumptions and constraints used in previously reported EMPC methods that may contribute to conservatism. Towards that goal the proposed algorithm has the following properties:

1 Terminal constraints (or periodic constraints) are not needed.
2 The cost is strictly an economic cost without additional terms (such as discounted costs used in previously reported studies) related to the deviations of the controlled and manipulated variables with respect to final optimal set points.
3 Robust stability is solved on-line using a polytopic model that captures model error. By solving the problem on-line potential conservatism that arises from the use of worst bounds in combination with worst inputs is avoided (Amrit et al., 2011; Diehl et al., 2011; Angeli et al., 2009; Rawlings et al., 2012).
4 Parameter updating is introduced to cope with parameter errors. Both the set point and the updating parameter value are introduced as additional decision variables in the economic optimization.

Since the proposed algorithm assumes the set-point to be a decision variable the need for an RTO calculation level is eliminated. The price for by-passing the RTO level is that stability has to be assessed with respect to a time varying set-point. Furthermore, a robust stability condition that accounts for model parametric uncertainty is computed and enforced online.

Towards that goal the EMPC algorithm is solved as an optimization problem over a receding horizon in which the nonlinear dynamic system behavior is represented by a polytopic model where its vertices are described by different operation points (Operation window) (Ellis et al., 2014; Huang et al., 2011; Kothare et al., 1996). A set of especial constraints is added so as to assure robust stability.

The organization of this paper is as follows.

In Section 2, the notation, assumptions, proposed algorithm and asymptotic robust stability are described. Simulation examples proving the performance of the algorithm are shown and discussed in Section 3 followed by conclusions in Section 4.

2. Definitions and methodology

2.1. Definitions and assumptions

Definition 2.1. (Positive definite function)

A function $\rho()$ is positive definite with respect to $\boldsymbol{x} = \boldsymbol{a}$ if:
It is continuous, $\rho(\boldsymbol{a}) = 0$ *and* $\rho(\boldsymbol{x}) > 0$ *for all* $\boldsymbol{x} \neq \boldsymbol{a}$

Definition 2.2. (Class $\mathfrak{K}$ function).

A function $\Upsilon() : \mathbb{R} \to \mathbb{R}_{\geq 0}$ is a class $\mathfrak{K}$ if it is continuous, zero at the origin and strictly increasing.

Lemma 2.1. Given a positive definite function $\rho(\boldsymbol{x})$ defined on a compact set D containing the origin, there exists a class $\mathfrak{K}$ function $\Upsilon()$ such that:

$$\rho(\boldsymbol{x}) \geq \Upsilon(|\boldsymbol{x}|), \quad \forall \boldsymbol{x} \quad \in \quad \mathbb{D}$$

Definition 2.3. (Positive invariant set).

A set $\mathbb{A}$ is positive invariant for the discrete nonlinear system $\boldsymbol{x}(k+1) = f(\boldsymbol{x}(k))$ if:

$$\boldsymbol{x}(k) \in \mathbb{A} \;\; and \;\; \boldsymbol{x}(k+1) \in \mathbb{A}$$

Definition 2.4. (Asymptotic stability).

The steady state $\boldsymbol{x}s$ of a nonlinear discrete system $\boldsymbol{x}(k+1) = f(\boldsymbol{x}(k))$ is asymptotically stable on $\mathbb{X}$, where $\mathbb{X}$ has $\boldsymbol{x}s$ in its interior, if there exist a $\Upsilon()$ such that for any $\boldsymbol{x} \in \mathbb{X}$, all solutions $\Phi(k;\boldsymbol{x})$ satisfy:

$$\Phi(k;\boldsymbol{x}) \in \mathbb{X},$$

$$|\Phi(k;\boldsymbol{x}) - \boldsymbol{x}s| \leq \Upsilon(|\boldsymbol{x} - \boldsymbol{x}s|, k) \quad \forall \mathrm{k} \in \mathbb{I}_{>0}$$

Where $\mathbb{I}_{>0}$ represents positive integers.

Definition 2.5. (Lyapunov function)

A function $V : \mathbb{R} \to \mathbb{R}_{\geq 0}$ is said to be Lyapunov function for the nonlinear discrete system in the set $\mathbb{X}$ if there exists $\Upsilon i()$, where $i \in \{1, 2, 3\}$ such that for any $\boldsymbol{x} \in \mathbb{X}$

$$\Upsilon 1(|\boldsymbol{x}|) \leq V(\boldsymbol{x}) \leq \Upsilon 2(|\boldsymbol{x}|);$$

$$V(\mathfrak{x}(k+1)) - V(\boldsymbol{x}(k)) \leq -\Upsilon 3(|\boldsymbol{x}|)$$

Lemma 2.2. (Lyapunov function and asymptotic stability). Consider a set $\mathbb{X}$ that is positive invariant for the nonlinear discrete system $\boldsymbol{x}(k+1) = f(\boldsymbol{x}(k))$. The steady state $\boldsymbol{x}s$ is an asymptotically stable equilibrium point for the system if and only if there exists a Lyapunov function V on $\mathbb{X}$ such that V satisfies the properties described above (Definition 2.5).

Assumptions.

i) The stage cost to be used by the EMPC algorithm and the nonlinear model are continuous.
ii) There is weak controllability. Therefore, there exists $\Upsilon() : \mathbb{R} \to \mathbb{R}_{\geq 0}$ so that for each $\boldsymbol{x} \in \mathbb{X}$ there exists a feasible $\boldsymbol{u}$ trajectory [u(1), u(2)...u(N)] with:

$$\sum_{k=1}^{N} |\boldsymbol{u}(k) - \boldsymbol{u}s| \leq \Upsilon(|\boldsymbol{x} - \boldsymbol{x}s|)$$

Where $\boldsymbol{u}s$ is the input vector corresponding to $\boldsymbol{x}s$.