Statistical mechanical modeling of the transition Stage II → Stage I of Li-ion storage in graphite. A priori vs induced heterogeneity

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ABSTRACT

An improved model to analyze the partial molar entropy step between stage II and stage I for lithium intercalation in graphite, based on statistical mechanics, is presented. At difference with our previous formulation, which assumed a priori heterogeneities, the present model considers the more realistic case of induced heterogeneities. The present work shows that, similarly to considerations made in the literature with intercalation cathode materials, most of the partial molar entropy changes for Li insertion into graphite may be explained on a configurational basis.

The interaction between ions intercalated in a same layer of graphite can be inferred to be strongly attractive, as was assumed in the literature for the simulation of voltammetric profiles.

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1. Introduction

Current lithium-ion batteries are largely based on graphite anodes and lithium metal oxide cathodes [1]. These materials allow a relatively fast ion insertion and extraction and therefore allow the fabrication of high power density cells, which have found ubiquitous applications. One of the important issues concerning the safety of these devices is heat generation, which may produce high temperature excursions of the cell and a premature aging of it or even hazardous events. Among the different types of heat, we can mention ohmic heat, heat exchange with the environment, heat of electrolyte precipitation and reversible heat [2], which can be estimated from the thermodynamics of the reactions involved. Thus, understanding thermodynamics aspects of the intercalation reactions allows to shed light onto one of the sources of heat in batteries, where the reversible heat generation rate may be found to be a significant portion of the total heat generation rate [3]. Moreover, partial molar entropy and enthalpy profiles have been found to show relevant changes with ageing of coin-cells batteries, which can be understood in terms of degradation of the crystal structure of electrode materials [4–6], thus providing a nondestructive tool for battery degradation analysis.

While thermodynamic properties on cathodic materials have been the subject of extensive experimental research [7–16] and modeling [17–22], the study of graphite anodes has been much more restricted. Experimental determination of enthalpy and entropy for this type of anodes has been achieved by Reynier et al. [23,24] and Yazami and Reynier [25]. In the case of Li-ion insertion into graphite, the experimental curve for entropy as a function of composition shows a step in the transition from stage II to stage I that could not be explained in the initial articles. In the third article [25], two of the previous authors stated that a possible mechanism of the stage I phase formation may involve a 'dilute lithium layer' (noted dil-Li) that would have alternating 'normal' Li layer (Li) with the hexagonal structure and a dilute lithium layer following the sequence (Li)–G–(dil–Li)–G. Pioneering modeling of Li insertion in graphite was undertaken by Derosa et al. [26] in terms of a lattice gas model. These authors simulated voltammograms, but did not consider partial molar properties. Computer simulation of partial molar enthalpy and partial molar entropy for this system were performed by Perassi et al. [27] showing that the transition from state II to state I involved the transition from an ordered state (II) to an initially disordered state. It is worth mentioning that Filhol et al. [28] constructed phase diagrams from first-principles calculations for this system, and focused on the absolute entropy rather than on the partial molar entropy. In a recent work [29], some of us proposed a simple two-levels lattice gas model that was able to account for the step in the partial molar entropy and concluded that attractive interactions must be assumed between adsorbates in a same lattice, to explain the half-width height of the

voltammetric peaks. In this former model, the occurrence of two energetic states involved an "a priori" heterogeneity, which is an oversimplification that may be easily modified within the same statistical mechanical approach. In the present work we explore an interaction model that introduces "induced" heterogeneity, which is found to be more adequate to describe the experimental features of the partial molar entropy.

2. Model and Statistical Mechanical background

Voltammetric experiments have shown that the occurrence of Stages I and II is related to two well defined energy states, which become evident as voltammetric peaks. In our previous work, we assumed the occurrence of two non-interacting lattices, where the lithium ions were hosted with different energies ("a priori" heterogeneity). In the present work, we consider a two layers supercell, made of two lattices, say I and II, as Fig. 1 shows. We will assume that these lattices may be occupied by particles (the Li^+ ions) with occupation numbers N_1 and N_2 respectively. The interaction energies of the particles within each one of the lattices will be assumed to be attractive, with the interaction parameter g. As mentioned in our previous work [29], there are both theoretical [28] and experimental [30] arguments supporting the hypothesis that this should be the case. Furthermore, the fitting of the voltammetric profiles corresponding to the formation of the different stages [30] suggests that they keep a similar interaction parameter for the particles being inserted in the same layer. On the other hand, the particles in the different layers will be allowed to interact with an effective pair repulsive energy Δ. This is also supported by the experimental fact that different diluted phases appear at low occupations of the system [31]. This model defines an induced heterogeneity. As in our previous formulation, particles within a same layer will be allowed to interact attractively. It will be assumed that each of the lattices has M sites where the incoming ions may be located, thus giving a total of $2M$ sites that may be occupied.

In statistical mechanics, all the thermodynamic properties of a system can be obtained from its partition function Q once it is known. In the canonical ensemble this is given by:

$$Q(N, 2M) = \sum_{i(states)} e^{-E_i/kT} = \sum_{j(levels)} \Omega_j e^{-E_j/kT} \quad (1)$$

where Ω_j labels the degeneracy of the jth energy level E_j and N the number of inserted ions. The term $2M$ indicates the total number of sites where the ions may be inserted, kT is the Bolzmann constant

multiplied by the absolute temperature and the sum run over all possible states that may be arranged with N particles. The degeneracy Ω_j may be straightforwardly computed counting the ways of distributing $N = N_1 + N_2$ particles among $2M$ sites with energy Ej.

According to the previous assumptions, for the present model we have:

$$E_j = N_1 E_0 + N_2 E_0 + gN_n \frac{N_1^2}{M}\frac{1}{2} + gN_n \frac{N_2^2}{M}\frac{1}{2} + \Delta \frac{N_1 N_2}{M} + \Delta \frac{N_2 N_1}{M} \quad (2)$$

where E_0 is the interaction energy with the graphite lattice (taken here as zero) and $N_n = 6$ is the number of neighbors in each of the lattices. From equation (2), it becomes clear that there is no a priori assumption of two different energy states, as it was supposed in our previous work.

We use a numerical procedure to evaluate the partition function Q for different situations, with the finding that results with $M = 100$ are converged with respect to the size of the system. Once Q was obtained for different values of N, different properties were obtained using the usual relationships between partition function and different thermodynamic properties [29]. We have analyzed voltammetric profiles, entropy, partial molar entropy and partial molar energy. We perform in all cases comparison with the results of our previous work.

3. Results and Discussion

Instead of using the number of particles as independent variable we define the fractional occupation X of the lattices as:

$$X = N/(2M) \quad (3)$$

so that all thermodynamic properties will be discussed in terms of this quantity.

Fig. 2 shows the entropy of the present system as a function of the occupation, for $g = 0$ and for different values of the parameter Δ/kT, which are indicated there. Comparison with Fig. 2 of our previous work shows that although increasing values of the repulsive factor Δ lead to two maxima in the entropy, as it was the case for increasing $\Delta E = E_2 - E_1$, the curves look different. Each of the two maxima become now more asymmetric, with the maxima values shifted to lower (higher) values when $X < 0.5$ ($X > 0.5$).
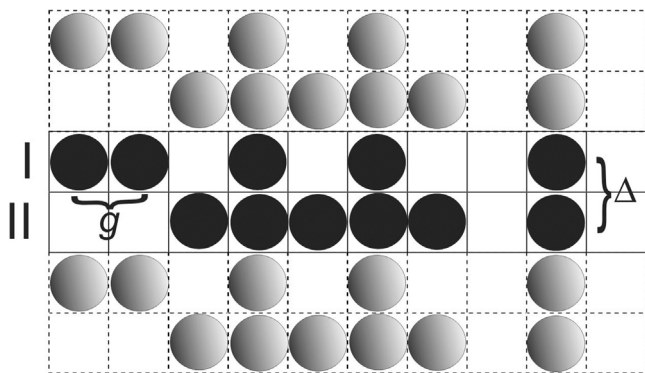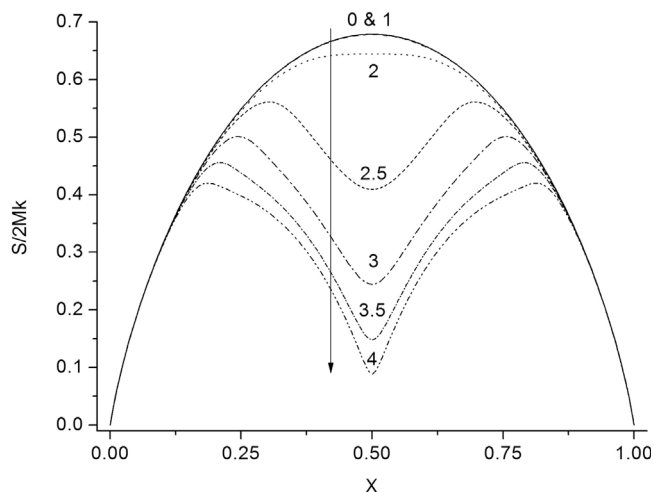

Fig. 1. Schematic representation of the present model for Li intercalation in graphite. The central part shows two partially filled slabs, say lattice I and lattice II, which may be progressively occupied by Li ions and are repeated periodically in space. The images are represented in light gray. The ions in the same slab interact with a pairwise attractive interaction g and the ions in the neighboring slabs with an effective interaction Δ. The interactions are averaged according to Eq. (2), as described in the text.

Fig. 2. Configurational entropy according to the model shown in Fig. 1, as a function of the fraction of occupied sites X. The entropy has been divided by the total number of lattice sites $2M$. The energetics of the system was described by Eq. (2), with $g = 0$. The Δ/kT values used are shown close to the plots. Δ/kT values where 0 (——), 1 (– –), 2 (····), 2.5 (– – –), 3 (– · – ·) 3.5 (– · · – ·) and 4 (– · · – · ·)respectively. The arrow indicates the behavior for increasing values of Δ/kT.